Lab 4: Compound Interest

Many banks compound interest periodically throughout the year. Just how much difference does it make if your bank compounds interest monthly while your friend's bank compounds interest daily? In this lab, we explore different strategies for compounding interest and consider the value of your principal under these different strategies.

Open the **Compound Interest Tool** in the Function Kit

- \( r \) represents interest rate, given in decimal form. To change the interest rate, move the \( r \) slider.
- \( n \) represents the number of times the interest is compounded each year. For example \( n = 12 \) means that the interest is compounded 12 times per year, or monthly.

The formula for the value of \( C \) dollars deposited in a bank paying an interest rate of \( r \) compounded times per year for \( t \) years is given by the function \( g(t) = C(1 + \frac{r}{n})^{nt} \). In this tool, we will let the deposit, \( C \) be $1.00, and the term be only one year, \( t = 1 \). (To find the value of \( C \) dollars under the same compounding, just multiply the answer by \( C \).)

1. **Constant interest rate with different compounding strategies**

   1.1 Using the sliders, set the interest rate to be \( .1, \) (10%). Now vary the number of times per year the interest is compounded. Complete the following chart:

   - value of $1.00 when interest is compounded monthly \( (n = 12) \) __________
   - value of $1.00 when interest is compounded weekly \( (n = 52) \) __________
   - value of $1.00 when interest is compounded daily \( (n = 365) \) __________

   1.2 Which method yields the best value after one year?

   1.3 If you had $1000 at the beginning of the year, how much would you have gained with daily compounding instead of monthly compounding?
2. Continuous compounding

2.1 If we were able to obtain interest every hour instead of every day, what would the value of \( n \) be?

2.2 If you were able to obtain interest at every minute, what would the value of \( n \) be?

2.3 If you could obtain interest at every second, what would the value of \( n \) be?

2.4 Use the slider to try larger values of \( n \) and observe what happens.

2.5 The number \( e \) is defined to be limiting value if you could obtain interest at every instant throughout the year. That is to say, the value of $1.00 would not grow without limit, but would approach the value of \( f(1) \). What is the approximate value of \( e \)?

3. Comparison of the Value of Principal over several scenarios (optional)

Generally, we are interested in finding out how different compounding strategies effect the value of our money at the end of a year. To do this, we can use the Compound Interest tool. With this tool, you can vary the interest rate and the compounding strategy.

Select the Function Kit and open the Compound Interest tool.
The value of \( g(t) \) represents compounding strategy of \( n \) times per year. If money is compounded continuously, the function \( f(t) \) will represent the value of the investment. In the table below, we compare different compounding strategies.

Complete the table below using the sliders to select the appropriate values of \( r \) and \( n \).

<table>
<thead>
<tr>
<th>( P )</th>
<th>( r )</th>
<th>( n )</th>
<th>( g(t) )</th>
<th>( f(t) )</th>
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</table>

Describe the effects of the components on the value of the investment:

a. Interest rate

b. Type of compounding