Instructor Notes 2: Families of Functions

We begin this lab with the following:

Functions are used in many ways in applications. One of these uses is to explain phenomena represented by data. Another use is to attempt to predict future outcomes (such as costs and sales) based on past history. As an example, knowing the present tuition and the pattern of tuition increases in the past, can we estimate what your tuition will be at your institution in five years. What does the pattern mean for future students?

Purpose:

To answer the questions above. In this lab, we are interested in two mathematical concepts:

I  The pattern exhibited by the phenomena, and
II  How a change in one parameter will affect future outcomes

We will explore both the behavior of a function that represents the phenomenon, and how a function changes when we have an impact on it, i.e., change its parameters.

The lab has three parts. Part A deals with common function families from high school algebra. Part B contains rational functions. Part C contains exponential function families. In the "Further Exploration" section, trigonometric functions are discussed.

Concepts developed:

The use of a parameter, an arbitrary constant, is explored. The behavior of a function \( f \) depends on the function, but also depends on the values of the constants, parameters, in the representation of the function. We explore the effect of the parameter on the behavior of many functions within the same family.

Students are asked to find common characteristics of functions in a family. Common features include patterns for roots, asymptotic behavior, existence of maximum or minimum points, etc.

Prerequisites:

Students should be familiar with functions. You might wish to use the Asymptote Tool to introduce students to the tools and the sliders, and the concept of asymptote. There are no specific prerequisites for this lab beyond an understanding of the concept of a function.

Tool Instructions:

Select the Function kit and choose the Families tool from the list.

To change the parameter, \( a \), move the slider with the mouse and click on the desired value. To determine what happens over time, roll the cursor left to right over the graph.

By rolling the cursor above the slider and clicking on various values, you can see overlays of the graphs for various values of the parameter, \( a \). Try this to see the general effect \( a \) has on the graph of the function.

If your students have studied the derivative at this point, they may verify the derivatives given.