There's no doubt that the "bottom line" is the maximization of profit, at least to the CEO and shareholders. However, the sales director might be more interested in maximizing sales and revenues. Questions such as how much to spend on advertising and/or price reductions arise naturally when we try to maximize revenue. When we try to maximize profits we must include cost-cutting strategies as well. If you were the CEO, what options would you choose, which technical expert would you believe?

1. The Demand Curve

The demand curve shows the relationship between the price per item and the quantity available in the market. We can look at it two ways. In the first case, the price is seen to determine the number of items sold at that price. Economists generally put the independent variable $p$ on the vertical axis for this interpretation and quantity $x = Q(p)$ on the horizontal axis. In the second case, which is usually used in calculus, we consider the quantity $x$ to be the independent variable and $p = D(x)$ is plotted on the vertical axis. Here we define:

$$p = D(x) = \text{the price per unit consumers are willing to pay}$$
$$\text{when } x \text{ items are available on the market.}$$

Demand curves generally have a certain look. They slope down as availability of the item increases. Occasionally they can be flat. A flat demand curve represents the case of pure competition where the individual producer is only one among a multitude of producers and whose effect on the quantity produced and hence the price is negligible.

1.1 Identify the curves shown with their most likely interpretation:

_____ a.) a linear demand curve (downward sloping)

_____ b.) a demand curve for pure competition

_____ c.) a non-linear demand curve

Figure 1
2. Revenue and Demand

The revenue is the quantity sold times the price per item. In terms of the quantity \( x \), we write:

\[ R(x) = xp = xD(x) \]

Analytically, in order to maximize the revenue we need to do two things. Initially, we need to find the critical points. These are the \( x \)-values where the tangent to the revenue function levels out (i.e., where the marginal revenue \( R'(x) = 0 \)) or where it is undefined (i.e., where \( R'(x) \) does not exist). Then we need to check that the revenue function \( R(x) \) is a maximum at that point.

2.1 Open the Cost/Revenue/Profit tool in the Economics Kit to see how these quantities are related graphically. At first we'll restrict our investigation to the revenue \( R(x) \) and marginal revenue \( R'(x) \) functions. Select these functions.

\[ R(x) = -5x^2 + 550x \]
\[ R'(x) = -10x + 550 \]

Also select the box for the tangent to the revenue curve. Roll the cursor over the upper graphing plane. What do you observe about the slope of the tangent line in this graph and the vertical coordinate of the point in the lower graph?

2.2 Find the \( x \)-value for which the tangent to the curve levels out. Is this value the same as the one you calculate by setting \( R'(x) = 0 \) and solving for \( x_{\text{crit}} \)?

What is that \( x \)-value?

What is the corresponding value of the revenue \( R \)?

From the graph, does it seem that this value of \( R \) is the maximum value?
2.3 Show by an analytical argument from your course that this value is the maximum revenue. (Hint: argue that the critical value gives a maximum from your knowledge of parabolas or from the First or Second Derivative Tests along with the fact that there is only one critical value.)

2.4 Let's look at the demand function \( p = D(x) \) for the given revenue function \( R(x) = -5x^2 + 550x \).

a. What is the demand \( p \) that corresponds to this revenue function?

b. On the same axes, sketch the demand function \( p = D(x) \) and the marginal revenue function \( R'(x) \). What are their relative slopes?

c. Show that it is always true that for any linear demand function \( p = ax + b \), the corresponding marginal revenue function is always a line with slope \( 2a \). (Note that \( a \) is expected to be negative.)

3. The Cost Function

The cost of producing \( x \) items is made up of two parts: a fixed cost and a variable cost. The **fixed cost** is the amount of cost involved in setting up the production such as rent, insurance, bookkeeping, etc. The fixed cost does not depend on the number \( x \) of items produced and would have to be paid even if no items are produced. The **variable cost** is a function of \( x \) and includes material and labor costs. Empirical evidence indicates that many cost curves have the appearance of cubic functions.
Open up the Cost/Rev/Profit tool.

3.1 Select the boxes so that the cubic cost function and the tangent to the cost function is displayed.

\[ C(x) = 0.2x^3 - 14.5x^2 + 450x + 1500 \]

a. What is the variable cost function?

b. What is the marginal cost function?

c. Roll the cursor over the upper graphing plane. What do you observe about the slope of the tangent line in this graph and the vertical coordinate of the point in the lower graph?

4. The Profit Function

Increases in revenue are exciting but if the cost increases excessively, the overall profit picture may be grim. A desirable profit picture can occur when the revenues are increasing much faster with an increase in production than the costs of production and distribution.

The profit from producing and selling \( x \) items is given by

\[ P(x) = R(x) - C(x) \]

Look again at the Cost/Revenue/Profit Tool. Now select the cost function \( C(x) \), the revenue function \( R(x) \) and the profit function \( P(x) \).

4.1 From the functions \( R(x) \) and \( C(x) \) given in the example on the tool, determine the profit function \( P(x) \). Algebraically determine the "break-even point." (It is the number \( x \) where \( C(x) = R(x) \), that is, where \( P(x) = 0 \).) Check your result with the tool.
4.2 Now select the marginal cost and the marginal revenue, as well as the tangents to the cost and revenue curves.

a. Describe what happens on the tool to the profit function $P(x)$ at the $x$-value where the marginal cost = marginal revenue ($C'(x) = R'(x)$).

b. Describe the relationship between the tangents to the revenue and cost curves at the $x$-value in part a.)

4.3 Use the tool to determine the $x$-values for which the revenue and profit are maximized:

For Revenue:

the maximum revenue is _____________ dollars at level of production

$x =$ ____________

For Profit:

the maximum profit is _____________ dollars at level of production

$x =$ ____________

4.4 The above relationship between maximum revenue and maximum profit is the usual one. Explain why in terms of the costs of production.