Lab 11: Taxes and the Optimization of Profit

What happens when a producer faces taxes? Do different kinds of taxes produce different results? How can a monopolistic producer change the level of production to maximize profits? Is it true that taxes raise prices to the consumer and discourage production? What about profits? If monopolist controls the market, can he or she simply pass all the additional costs due to taxes to the consumers and maintain profits? Let's find out!

For this lab, we are restricting our attention to monopolists, because their cost, revenue and profit functions are the same as those for the entire industry. Consequently, their decisions unilaterally affect the entire industry. We're going to look at the effect of taxes on profit and the optimum pricing strategies for a monopolist.

Note that for this example the demand curve is linear. The results in this lab follow from the assumption that the demand function is always a decreasing function. In our particular case the slope of the linear demand function is negative.

1. Unit taxes on the producer

On the Cost/Revenue/Profit tool in the Economics Kit, select the unit tax option. A unit tax \( t \) is a fixed tax on each item produced. Because the tax is applied for each item produced, the additional cost is \( (\text{where } x \text{ denotes the quantity}) \) and of course there is an additional marginal cost \( t \).

Note that a tax slider appears. You can get a sequence of curves by putting the cursor on the numerals above the slider, rather than directly on the slider. It's really quite spectacular and gives you the ability to compare the effects of changes in the tax. You can clear away the curves by moving your cursor to the slider below the numerals.

1.1 To see how this kind of tax affects the profit, select values on the upper part of the tax-slider to generate a sequence of cost curves for various values of the unit tax \( t \). Try equal increments of the tax \( t: 0, 50,100,150,... \)

a. Describe what happens to the marginal cost curves as the tax \( t \) is increased in equal increments.
b. Describe what happens to the profit curves and the corresponding maximum profits as the tax \( t \) is increased in equal increments.

c. Find the unit tax \( t \) that produces zero profit both experimentally and algebraically. (Of course at that point the monopolist loses interest in producing any items at all!)

d. However for a reasonable tax, say \( t = 50 \), what should the monopolist do to maximize profit. Show \textit{algebraically} that the monopolist should reduce production and raise his price. In fact, show that he should pass on exactly half of the additional cost to the consumer by raising his price and by absorbing the remaining half. (It can be proved that for a linear demand function and a unit tax, that this is always the best strategy to maximize profits.)

2. The effect of a sales tax on profits

Again we restrict our attention to monopolists, since their decisions are the responses of the respective industries which they control.

The effect of a sales tax is somewhat more subtle than a unit tax. A sales tax raises the price to the consumer without raising the revenue of the producer. Consumers respond according to their demand function.

Let \( t \) be the fixed sales tax and \( p = D(x) \) be the price corresponding to a zero sales tax.
Note that the price per item that the consumers see is $p_{\text{consumer}} = p_{\text{producer}} + t p_{\text{producer}}$ which lowers their demand according to their demand curve $p = D(x)$ and gives a net price to the producer of

$$p_{\text{producer}} = \frac{p_{\text{consumer}}}{1+t} = \frac{D(x)}{1+t}.$$ 

2.1 What is the corresponding revenue to the producer in terms of $D(x), x$ and $t$?

Now select the sales tax option on the Cost/Revenue/Profit tool. To see how this kind of tax affects the profit, use the upper part of the tax-slider to generate a sequence of cost curves for various values of the sales tax $t$. Try fairly even increments of the tax $t$: 0.20, 0.40, 0.60,... Remember to use the upper part of the slider (where the numerals are) to get overlays of the curves.

2.2 What is happening to the slope of the marginal revenue curve $y = R'(x)$ as the tax $t$ is increased?

2.3 In order to understand why the marginal revenue is changing, note that the demand curve is changing also. Look at the lower graphing plane. Simplify the picture by choosing $t = 0$ and $t = 30\%$.

Which demand function is the one that determines the purchases by the consumers (i.e., upper, lower)?

Which demand function is the one that affects the producer (i.e., upper, lower)?
2.4. Describe what is happening to the profit and the optimum level of production $x$ (that maximizes the profit) as the tax $t$ is increased.

2.5 By experimenting with the $t$-slider, find the lowest tax for which it is no longer possible for the monopolist to make a profit at any level of production $x$.

3. Further Considerations

We will examine the effects of taxes on the welfare of the producers and consumers, as well as governments, after the definite integral has been introduced.