Lab 9: Optimizing Costs

One method for improving profits is to lower costs. Cost containment, restructuring and leaner, more competitive companies, along with increases in productivity, have led the way to the dominance of US. companies at the turn of the century. What will happen next? Let's look at some of the ways to optimize costs.

In this lab we will discuss a variety of cost functions.

1. The Total Cost Function

The total cost function $C(x)$ is the cost of producing a quantity $x$ of goods.

Many times cubic functions are used to model total cost. Occasionally linear or quadratic cost functions are used. Cost functions are made up of fixed costs and variable costs. Fixed costs are those costs that are incurred even if no items are produced, for instance, rent, executive salaries, secretarial and bookkeeping services, etc. Variable costs are dependent on the number $x$ of items produced.

Cubic cost functions $y = C(x)$ are selected that have the following properties:

- The cost function must be an increasing and positive-valued function: $C(x)$ and $C'(x) > 0$ for $x > 0$.

- The slope of the tangent line to the cost function must be large at first when only a few items are produced, then it becomes less steep when returns to scale are realized and then becomes steep again when too many items are being produced for efficient production.

1.1 Select the cubic function that has both the required properties from the graphs in Figure 1. Explain why you did not select the other cubic functions.

![Figure 1]
1.2 Open the **Cost Curves** tool in the **Economics Kit**. Move the $a$-slider to generate a family of cubic cost curves. Set the $a$-slider to $a = 1$. Do all of the curves generated have these properties?

2. **The Marginal Cost Function**

The **marginal cost function** is the derivative of the total cost function:

$$MC(x) = C'(x)$$ where $x$ is the quantity produced.

Remark: The marginal cost is roughly the cost of producing an additional item at the given production level $x$.

2.1 Now select the box for the **Tangents to the Cost Curve**. Move the cursor across the top screen to see the tangent slide along the total cost curve. Describe how the tangent is related to the marginal cost curve in the lower screen.

2.2 Let's look at some other total cost curves $y = C(x)$ to see if you have the right idea. Curve A is a linear cost curve. Curve B is a suitable part of a quadratic function (i.e., the minimum is in the negative half-plane).

On the axes below the given graphs, sketch graphs of the marginal cost functions for A and B.

![Figure 2](image)
3. The Average Cost Function

The average cost function is the quotient of the total cost function $C(x)$ and the quantity produced $x$.

$$AC(x) = \frac{C(x)}{x}$$

A monopolist can decide to produce less for a higher price because he has control over the total output. However under pure competition, producers are not able to affect the price for what they produce. If there is a significant difference between the average cost and the price in a free market, more producers will move into production which will drive down the price.

Gradually profits are squeezed and soon approach the minimum average cost for the producers concerned. Most firms operate in industries that are somewhere between monopolies and pure competition. There is a natural tendency to consolidate in order to control more of the market.

3.1 On the Cost Curves tool, set the $a$-slider to $a = 1.5$. Select the box for the Average Cost Secant. As you move the cursor over the upper graph, notice the blue line. This line connects the point on the total cost curve with the origin. When $y = C(x)$, what is the slope of the blue line?

$$m = \frac{\text{rise}}{\text{run}} =$$

3.2 With the $a$-slider set to 1.5, find the value of $x$ at which the average cost is minimized. What is the average cost at that $x$-value?

3.3 Observe the marginal cost and the average cost curve on the tool. Is the average cost minimized at the production level $x$ where the marginal cost is equal to the average cost? (If you don't see this immediately, try other values on the $a$-slider and look for a pattern.)
3.4 Think about why the production level \( x \) where the average cost equals the marginal cost is the one for which average cost is minimized. (This question requires insight. Here are some "leading questions").

If the marginal cost is less than the average cost, then the average cost is ______________ (reduced, increased) by increasing the level of production \( x \).

If the marginal cost is greater than the average cost, then the average cost is ______________ (reduced, increased) by increasing the level of production \( x \).

3.5 Notice that in both cases in 3.4, the average cost is reduced as \( x \) approaches (from either side) the value at which marginal cost equals average cost. Look at how the average changes and recall that marginal cost is roughly the cost of producing the next item. Use these ideas to explain why the average cost is minimized at the value of \( x \) where the marginal cost equals the average cost.

Explain:

Label the two curves "average cost" and "marginal cost"

![Figure 3](image)
3.6 Use the definition of the average cost (1) and the quotient rule for differentiation to show that the critical point for the average cost occurs at the \( x \)-value where the average cost equals the marginal cost.

\[
\frac{d}{dx} (AC(x)) =
\]

4. Variable Cost and Average Variable Cost

The variable cost function \( V(x) \) is the total cost \( C(x) \) minus the fixed cost \( F(x) \). (Recall that \( F(x) \) is a constant.)

\[
V(x) = C(x) - F(x)
\]

The average variable cost function is the quotient of the variable cost function \( V(x) \) and the quantity produced \( x \).

\[
AV(x) = \frac{V(x)}{x}
\]

4.1 Select the box for the variable cost on the Cost Curves tool. Describe what happens at the point where the average variable cost is at a minimum. (The situation should seem familiar!)

4.2 Go back to Figure 3 and sketch in the variable cost curve. Be certain that the minimum is placed properly.
4.3. Use the definition of the variable cost $V(x)$, the average variable cost $AV(x)$ and the quotient rule for differentiation, to show that the critical point for the average variable cost function always occurs at the production level $x$ where it intersects the marginal cost function $MC(x)$.

Note that you will need the fact that $C'(x) = V'(x)$. Why is this true?

\[
\frac{d}{dx} (AV(x)) =
\]

5. Concluding remarks

The tendency of free markets to attract new producers when the price is greater than the minimum average cost tends to drive down profits and to enhance the buying power of the consumer. There is a natural tendency among producers to act in concert in order to control the market. And of course there are safeguards but usually the greatest inhibitor of collusion is the free substitution of alternate products.