Constraints
Constraints and valuations

- **Constraint Domain.**

- **Function symbols:** in $\mathbb{R}$ we have $\dagger, -, \times$ and $/$;

- **Constraint relation symbols:** in $\mathbb{R}$ we have $\equiv, <, \leq, >, \geq$

- **Primitive Constraints**
Satisfying Constraints

• Valuation $\theta$: Assignment of values in the domain to variables in the constraint $C$;

• Solution: A valuation $\theta$ for $V$ including $V(C)$ such that $\theta(C)$ holds in the constraint domain.
Equivalence of Constraints

Two constraints $C_1$ and $C_2$ are equivalent if they have the same set of solutions.

The questions of satisfiability and equivalence are critical for constraint programming as we shall see.
Modeling with Constraints

- Electric Circuits

- Job Scheduling
Constraint Satisfaction

- Enumeration
- Linear Constraints
- Gauss Jordan Elimination
Tree Constraints

A tree constructor is a string of characters beginning with a lower case character.

A tree is defined recursively: A constant is a tree, and a tree constructor together with an ordered list of \( n \geq 1 \) trees, called its children is a tree.

Finite Trees are tree with finite height.
Terms

A *term* is defined as follows: It is either a constant, or a variable, or a tree constructor together with an ordered list of $n \geq 1$ terms.

- *Term equation*

- *Tree constraint*
Tree Constraint Solver
Unification

unify($C'$)

$S := \text{true}$

while $C$ is not empty

let $C$ be of the form $c \land C_0$

\textbf{cases}

(1) $c$ is of the form $x = x$:

$C := C_0$

(2) $c$ is of the form $f(s_1, \ldots, s_n) = g(t_1, \ldots, t_m)$

where $f \neq g$ or $n \neq m$:

return false;

(3) $c$ is of the form $f(s_1, \ldots, s_n) = f(t_1, \ldots, t_n)$:

$C := (s_1 = t_1) \land \ldots (s_n = t_n) \land C_0$
(4) $c$ is of the form $t = x$ where $t$ is not a variable:
   
   $$C := (x = t) \land C_0$$

(5) $c$ is of the form $x = t$ where $t$ contains $x$:
   
   return false

(6) $c$ is of the form $x = t$:
   
   substitute $t$ for $x$ throughout $C_0$ and $S$
   
   $$C := C_0$$
   $$S := S \land c$$

end cases

end while

return $S$