Finite Constraint Domains

A constraint satisfaction problem, (CSP), consists of a constraint $C$ over variables $x_1, \ldots, x_n$ and a domain $D$ that maps each variable to a finite set of values, $D(x_i)$.

$$C \land x_1 \in D(x_1) \land \ldots \land x_n \in D(x_n)$$
Examples

- Map colouring

- $N$-queens

- The marriage problem

Solution by backtracking – uses satisfiable($c$) which takes a primitive constraint with no variables and returns true or false according to satisfiability.
back_solve(C,D)
    if \( \textit{vars}(C) \) is empty then
        return \( \textit{partial_satisfiable}(C) \)
    else
        choose \( x \in \textit{vars}(X) \)
        for each value \( d \in D(x) \) do
            let \( C_1 \) be obtained from \( C \)
            by replacing \( x \) by \( d \)
            if \( \textit{partial_satisfiable}(C_1) \) then
                if \( \textit{back_solve}(C_1, D) \) then
                    return \( \text{true} \)
                endif
            endif
        endfor
        return \( \text{false} \)
    endif
endif
partial_satisfiable(C)
  let $C$ be of the form $c_1 \land \ldots \land c_n$
  for $i := 1$ to $n$ do
    if $\text{vars}(c_i)$ is empty then
      if $\text{satisfiable}(c_i) = \text{false}$ then
        return $\text{false}$
      endif
    endif
  endif
endfor
return $\text{true}$
Doing Better

• A primitive constraint $c$ is *node consistent* if either $|\text{vars}(c)| \neq 1$ or, if $\text{vars}(c) = x$, then for each $d \in D(x)$, \{$x \rightarrow d$\} is a solution of $c$. Also for a CSP.

• A primitive constraint $c$ is *arc consistent* if either $|\text{vars}(c)| \neq 2$ or, if $\text{vars}(c) = x, y$, then for each $d_x \in D(x)$ there is a $d_y$ such that \{$x \rightarrow d_x, y \rightarrow d_y$\} is a solution of $c$. Also for a CSP.
node_consistent(C,D)

let $C$ be of the form $c_1 \land \ldots \land c_n$

for $i := 1$ to $n$ do

\hspace{1cm} D := node_consistent_primitive(C_i, D)

endfor

return $D$
\begin{verbatim}
node_consistent_primitive(c, D)
    if | vars(c) |\(\leq\) 1 then
        let \(\{x\} = vars(c)\)
        \(D(x) : -\{d \in D(x) | \{x \rightarrow d\} \text{ is a solution of } c\}\)
    endif
    return \(D\)
\end{verbatim}
arc_consistent

let $C$ be of the form $c_1 \land \ldots \land c_n$

repeat

$W := D$

for $i := 1$ to $n$ do

$D := \text{arc_consistent_primitive}(C_i, D)$

endfor

until $W = D$

return $D$
arc_consistent_primitive(c, D)

if | vars(c) | \(\geq\) 2 then

let \(\{x, y\} = vars(c)\)

\[ D(x) := \{d_x \in D(x) | \text{for some } d_y \in D(y), \{x \to d_x, y \to d_y\} \text{ is a solution of } c\} \]

\[ D(y) := \{d_y \in D(y) | \text{for some } d_x \in D(x), \{x \to d_x, y \to d_y\} \text{ is a solution of } c\} \]

endif

return \(D\)
Arc Consistency Solver

\textbf{arc\_solve}(C, D)

\[ D := \text{node\_arc\_consistent}(C, D) \]

\textbf{if} \( D \) is false \textbf{then}

\textbf{return} false

\textbf{elseif} \( D \) is a valuation domain \textbf{then}

\textbf{return} satisfiable\((C, D)\)

\textbf{else}

\textbf{return} unknown

\textbf{endif}
node_arc_consistent(C,D)

\[
D := \text{node\_consistent}(C, D)
\]

\[
D := \text{arc\_consistent}(C, D)
\]

\text{return} \ D
back_arc_solve(C,D)

\[ D := \text{node.arc.consistent}(C, D) \]

if \( D \) is false then

return false

elseif \( D \) is a valuation domain then

if satisfiable\((C, D)\) then

return \( D \)

else

return false

endif
choose $x$ such that $|D(x)| \geq 2$
for each $d \in D(x)$
\[
D_1 := \text{back_track_solve}(C \land x = d, D)
\]
if $D_1 \neq false$ then
\[
\text{return } D_1
\]
endif
endfor
return false