Dynamics of defects and traveling waves in an interfacial finger pattern

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Abstract

We present the results of an experimental study of one-dimensional traveling finger patterns dominated by source and sink defects. The system studied is a driven fluid–air interface, and the traveling-wave state arises via a subcritical bifurcation. We analyze space–time images of the patterns to obtain the amplitude and local wave number of the left- and right-going traveling waves near the defects. We determine the width of the defects, and find that the width of the sinks does not vary significantly with distance above the bifurcation over the range of our experiments, while the source width increases as the bifurcation is approached from above. We also observe both localized depressions or enhancements in wave amplitude and periodic modulations of the pattern amplitude and wave number.

Keywords: Defects; Traveling waves; Instabilities; Pattern formation; Fingering

1. Introduction

Source and sink defects have been observed in a variety of one-dimensional pattern-forming systems with traveling-wave states [1–10]. These defects separate regions of oppositely moving traveling waves: sources emit left- and right-going waves, while sinks absorb them. They are the fundamental defects of one-dimensional traveling-wave patterns, and an understanding of their behavior is key to understanding the dynamics of more complex spatio-temporal systems.

Sources and sinks have been studied theoretically in the context of coupled complex Ginzburg–Landau equations (CCGLE) [11]. The properties of sources and sinks in travelling-wave patterns which develop above a supercritical Hopf bifurcation have been analyzed by several groups [12–17]. Van Hecke et al. carried out a
detailed study of defects in solutions to CCGLE [16]. Their simulations indicated that sources control the dynamics of the pattern. Sources were found to be stable and symmetric with respect to the wave number of the pattern emitted on either side of the source above a certain critical value of the control parameter $\epsilon$. Below this value the control parameter sources become unstable and non-stationary [12] and start to “breathe”, that is, their width varies with time. This transition in defect behavior is associated with the transition from convective to absolute instability of the travelling-wave state [18–21]. Sinks, on the other hand, are essentially passive. The wave number of the pattern on the two sides of a sink can be different, and their behavior did not change as a function of $\epsilon$. The widths of both sources and sinks were found to diverge as $\epsilon^{-1}$ close to the supercritical Hopf bifurcation at $\epsilon = 0$ [16]. Coherent structures in the wave pattern have also been studied theoretically. So-called holes in the travelling-wave pattern – that is, localized depressions in the amplitude of the wave, coupled with a change in the wave number – have been observed in solutions to CCGLE [17,22–24]. Nozaki-Bekki holes [22] connect regions of traveling waves with different wave number, while homoclinic holes or homoclinic holes [17,23,24] connect regions with the same wave number. Modulated amplitude waves, which are coherent traveling modulations in both amplitude and wave number, have been studied recently [17,25–27]. Although both holes and modulated amplitude waves tend to be linearly unstable, they nonetheless play an important role in the development of spatio-temporal chaos and complex dynamics in solutions of the CCGLE.

Experimentally, sources and sinks have been studied extensively in traveling-wave convection driven by a heated wire [1–3,21], as well as in other systems [4–9]. Traveling waves in heated-wire convection appear via a supercritical Hopf bifurcation, as in the CCGLE. Alvarez et al. [2] found sources to be stationary and symmetric – that is, the wave number emitted was the same on each side of the defect – but different sources emitted waves of different wave number. They also found that sinks sandwiched between two patches with different wave numbers moved such that the phase difference across the source remained fixed. This phase-matching behavior does not appear in solutions to the CCGLE, which do not treat the short length scales involved. The experiments of Garnier et al. [21] showed a logarithmic variation in source width which was consistent with predictions for the behavior of defects near the convective-absolute instability transition [19]. Pastur et al. [3] found that below a critical value of the control parameter $\epsilon$, sources started to fluctuate strongly and their width diverged as $\epsilon^{-1}$. Sinks moved according to the above phase-matching rule, and their width did not diverge as $\epsilon$ was decreased. They observed homoclinic holes in the pattern which were emitted from sources and propagated through the pattern, and observed holes connecting regions of different wave number in transient patterns when the driving force was changed. Additional experimental evidence for both holes [28–30] and modulated amplitude waves [31–34] in other traveling-wave systems has recently been reviewed by van Hecke [17].

The printer’s instability (also called directional viscous fingering) [10] is a system well suited to the study of the dynamics of one-dimensional patterns in extended systems [35]. The system consists of a fluid–air interface confined in the gap between two acratically mounted horizontal cylinders. When the system is driven away from equilibrium by rotating the cylinders, a variety of fingering patterns can develop on the interface. Among the phenomena which have been studied in this system are stationary finger patterns [10,36], traveling finger patterns [8,10,37–39] which arise from a parity-breaking instability [37,38,40,41], spatio-temporally chaotic patterns [8,10,42], and sink and source defects [8–10] which separate regions of oppositely traveling fingers. Habdas et al. [9] studied the behavior of sink and source defects in this system. They found that sinks separate regions of differing wave number and move with the phase-matching behavior described above. Sinks were observed to be transient objects which eventually annihilated by collision with a source or the boundaries of the system. Isolated sources, on the other hand, persisted for the duration of the experiment. They were symmetric and stationary on average, although individual sources underwent small-scale random motion apparently driven by noise.

In this paper we describe the results of a study of the amplitude and local wave number near source and sink defects in the printer’s instability. Using Fourier decomposition, we analyze the amplitudes of the counterpropagating waves on each side of the defects. From this analysis we determine the width of the defects as
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a function of the experimental control parameter. We also describe features similar to the propagating holes and the modulated amplitude waves mentioned above, and discuss how these features affect the behavior of the defects. In this system the traveling pattern develops via a parity-breaking transition[37,38,40,41] rather than a Hopf bifurcation, and in the work reported here the transition is subcritical. Despite these differences, aspects of the phenomena we observe are nonetheless similar in many ways to those seen in systems more closely described by the CCGLE, suggesting that aspects of the defect behavior are independent of the origin of the travelling-wave state.

2. Experimental

The apparatus used in these experiments is described in detail in our previous papers[8,9,36–38], and shown schematically in Fig. 1. It consisted of two horizontal cylinders mounted one inside the other. The inner cylinder was made from white Delrin and had a radius of 38.7 mm and a length of 202 mm. The outer cylinder was made from Plexiglas and had a radius of 66.7 mm and a length of 210 mm. The axes of the cylinders were parallel but offset vertically so that the gap between the cylinders was smallest at the bottom of the apparatus. In these experiments, \( b = 0.50 \pm 0.05 \) mm. Silicone oil (viscosity 0.525 g/cm s, surface tension 19.4 g/s², and density 0.963 g/s³[43]) was poured into the gap. Our results were not affected by the quantity of oil, as long as it was sufficient to keep the narrowest part of the gap filled. The two cylinders could be rotated independently. The finger patterns which formed at the oil–air interface were monitored at the front of the apparatus with a ccd video camera interfaced to a computer. The spatial uniformity of the finger patterns is very sensitive to misalignment of the cylinder axes, and we ensured that the axes were accurately parallel by inspection of the patterns.

When the outer cylinder was held fixed and the inner cylinder rotated so that the bottom of the cylinder approached the camera, stationary fingers appeared on the oil–air interface at a specific value of the inner cylinder surface velocity \( v_i \). For our geometry, the bifurcation to fingers occurs at \( v_i = v_{ic} = 197 \) mm/s and is subcritical[36,38]–the fingers appeared with a finite amplitude and disappeared at \( v_i < v_{ic} \). The stationary pattern undergoes a bifurcation to a traveling finger state when the outer cylinder is rotated in the opposite direction to the rotation of the inner cylinder. We set the outer cylinder velocity to \( v_o = 22.75 \) mm/s and studied the patterns as a function of \( v_i \). Within our experimental resolution, the onset of fingering did not change from \( v_{ic} \) for this small value of \( v_o \); we thus used \( \epsilon = (v_i - v_{ic})/v_{ic} \) as our dimensionless control parameter. In the experiments reported here \( 0.09 \leq \epsilon \leq 0.70 \). At lower \( \epsilon \) defects were much less stable and did not survive long enough to be studied. Note that for this experimental trajectory, the transition to traveling waves was slightly subcritical, in contrast to the case studied in Refs.[8,37,38].

Sources and sinks were created with a rubber wiper attached to a rod which was used to locally disturb the layer of fluid coating the inner cylinder. This in turn perturbed the finger pattern on the oil–air interface. This method produced both sources and sinks over a range of \( \epsilon \). Typically the perturbations produced several transient defects in the finger pattern which rapidly disappeared through collisions with other defects or with the end of the apparatus. The sources and sinks studied in this paper are all longer-lived defects which persisted after the initial transient variations had settled down.

3. Results

Fig. 2 shows a space–time image of a source defect at \( \epsilon = 0.185 \). This image was constructed by periodically

Fig. 1. A diagram of the experimental apparatus viewed from the front. The outer and inner cylinders are horizontal with their axes accurately parallel, but vertically offset. The outer cylinder rests on rollers while the inner cylinder is supported by an axle as shown. The gap between the two cylinders has a minimum value of \( b = 0.5 \) mm and broadens towards the front and back of the apparatus (out of and into the page in this diagram). Oil is placed in the gap and the cylinders move in counter-rotation as indicated by the arrows. The fingering pattern on the meniscus is viewed from the front.
Fig. 2. A space–time image of a source defect at $\varepsilon = 0.155$. Position runs horizontally with the field of view being 125 mm, 62% of the length of the inner cylinder. Time runs from top to bottom; the image covers 20 s. The bright regions are fingers in the oil–air interface which propagate away from the source. The black line running from top to bottom is the location of the defect as determined using the method described in the text.

recording a single video line through the finger pattern, then stacking these line images vertically. Position $x$ along the length of the experiment runs horizontally and time $t$ runs vertically from top to bottom. The amplitudes of the left- and right-going traveling waves emitted from this source are shown in Fig. 3. These were calculated by Fourier decomposition of the space–time image. The image was apodized, then Fourier transformed. Peaks corresponding to the two directions of traveling wave were isolated and shifted to zero frequency. The resulting spectra were low-pass filtered to remove higher harmonics of the wave pattern, then inverse transformed to give the amplitudes $A_L$ and $A_R$ of the fundamental frequency components of the left- and right-moving waves as functions of $x$ and $t$. Fig. 3 is actually a plot of $A_R - A_L$, color coded so that regions dominated by the right-moving wave are red and those dominated by the left-moving wave are blue.

The local wave number $k(x, t)$ for the same defect is shown in Fig. 4 [9,44]. To calculate $k(x, t)$, the space–time image was first bandpass filtered in Fourier space to remove all but the fundamental spatial and temporal frequencies. Both the frequency and the wave number of the traveling waves were then calculated from appropriate derivatives of the filtered real-space images, as described in Ref. [44]. The wave number map shown in Fig. 4 has been smoothed and plotted with a grayscale which enhances the features of interest to the present work; this makes the region close to the defect itself, where $k$ is slightly larger than in the rest of the pattern, appear white. The outward-moving, periodic modulations of $k$ are real, but higher-frequency features which remain (for example at the upper left of the figure) are not robust and are artifacts of the image processing. $A_L$, $A_R$, and $k$ are shown in Fig. 5 for the particular time indicated by the horizontal lines in Figs. 2–4.
Fig. 3. A space–time map of the amplitude of the left- and right-moving waves emitted from the source of Fig. 2. The colors indicate the value of $A_R - A_L$, with red indicating that $A_R$ dominates and blue the $A_L$ dominates. The black line running from top to bottom is the location of the defect as in Fig. 2. The two wave amplitudes are plotted at the time corresponding to the horizontal line in Fig. 5.

We define the location of the defect $x^*$ to be at the point at which the amplitudes of the two waves are equal, $A_L(x^*) = A_R(x^*) = A^*$. This position corresponds closely to the position one would determine by eye from a space–time plot such as Fig. 2 in most cases. Software locates and tracks the defect through each space–time image semi-automatically; the result for the defect of Figs. 2 and 3 is shown as the roughly vertical black line in those figures. In cases where the defect is less “well-behaved” our software is unable to track its position accurately; some of the reasons for this are discussed below.

We take the width of the defect to be the distance between the points $x_L$ and $x_R$ at which the amplitudes of the left- and right-going waves drop to $A^*/e$, $e$ being the base of natural logarithms. The width is then $w = |x_R - x_L|$, where the absolute value is needed when we apply the same definition to sinks. $\ell_L = |x_L - x^*|$, and $\ell_R = |x_R - x^*|$ are effective length scales for the growth of the two counterpropagating waves. For purely exponential growth $\ell_L$ and $\ell_R$ would be simply the exponential growth lengths of the two waves and $w$ their sum. In the present case $A_L$ and $A_R$ in fact do grow approximately exponentially with position near sources when the amplitudes are small, but they are often beyond the exponential growth regime at the point where the two amplitudes are equal. They are also substantially affected by dynamical phenomena taking place at the defect, such as the emission of the amplitude modulations mentioned above. This will be discussed further below.

Fig. 6 shows the mean width $\langle w_s \rangle$ of the sources as a function of $\epsilon$. Each point on the graph is an average over several measurements, and each individual measurement $w_i$ is itself determined by averaging the source width over a significant time interval, ranging from about 3 to 25 periods of the traveling waves. The error bars are the mean values of the standard deviations.
of the $w_s$ and are a measure of the variability of $w_s$ with time. Simply using the standard deviation of $\langle w_s \rangle$ gives similar but slightly smaller error bars. The mean width of the sources appears roughly constant for $\epsilon > 0.3$ but increases steadily as $\epsilon$ decreases below this value.

The inset to Fig. 6 shows $\langle w_s \rangle$ plotted as a function of $1/\epsilon$. The data are consistent with a linear increase in $\langle w_s \rangle$ with $1/\epsilon$, starting from a finite value at $\epsilon = \infty$: a fit gives $\langle w_s \rangle = (15.6 \pm 0.6) + (0.98 \pm 0.15)/\epsilon$ mm. Unfortunately we were unable to generate long-lived sources at lower $\epsilon$, so, while our data are suggestive, we are unable to say whether or not $\langle w_s \rangle$ diverges as $\epsilon \to 0$. $\ell_L$, the length scale for the growth of the left-going wave near the source, also shows a weak increase as $\epsilon$ decreases below about 0.3. $\ell_R$, on the other hand, is approximately constant for all but the lowest $\epsilon$ point, for which it increases significantly. On average, the length scale for the growth of the left-going waves is about 40% larger than for the right-going waves.

The variability of the source width, as measured by the error bars plotted in Fig. 6, is roughly constant over the range of $\epsilon$ studied, except for the data at the lowest value of $\epsilon$, $\epsilon = 0.094$. For this value of $\epsilon$ the variability of the source width is approximately twice as large as, and the dynamics of the source is qualitatively different from that at higher $\epsilon$. Fig. 7 is a space–time image of the source at $\epsilon = 0.094$. It is much less stable than the sources at higher $\epsilon$, such as that in Fig. 2, in the sense that its position drifts about the apparatus much more, and the pattern as a whole appears less ordered. (The mean pattern wave number is also larger than at higher $\epsilon$, as expected close to the onset of fingering [36].) The pattern away from the source appears much more sensitive to perturbations, and, as can be seen in Fig. 7, perturbations in $k$ actually can propagate into the source. It is also worth noting that at lower $\epsilon$, sources were sufficiently unstable that they did not survive long enough for us to study them quantitatively.
Fig. 5. The solid and dashed lines are the amplitudes of the left- and right-moving traveling waves, respectively, emitted from the source of Figs. 2–4 at the time indicated by the horizontal lines in those figures. The dotted line is the local wave number of the pattern at the same time. We define the position of the defect to be the point at which the two amplitudes are equal. Note the correspondence between the modulations of amplitude and wave number visible on the right-hand side of the figure.

The same analysis was applied to sink defects. Fig. 8 is a space–time diagram of a sink drifting steadily to the left; in this case the propagating fingers travel into the defect where they are absorbed. The amplitudes of the left- and right-moving traveling waves are shown in Fig. 9, which again shows $A_R - A_L$ as a function of $x$ and $t$. The sink separates regions of different wave number, as indicated by the grayscale shading, and moves towards the region of lower $k$ [9]. $A_R(x)$, $A_L(x)$ and $k(x)$ are plotted in Fig. 10 for the time indicated by the horizontal line in Figs. 8 and 9.

As before we define the position of the defect as the point where the wave amplitudes $A_L$ and $A_R$ are equal, and the width as the distance between the points where the amplitudes drop to $1/e$ of their value at the defect. In this case, however, the amplitudes decrease roughly linearly in the region of the sink, not exponentially.

The position of the sink as determined by our software is shown as the roughly vertical line in Figs. 8 and 9, and again agrees reasonably well with the position determined by eye. The mean width ($w_s$) of the sink defects as a function of $\varepsilon$ is plotted as open circles in Fig. 6. The error bars are again based on the mean value of the standard deviation for all measurements at a given value of $\varepsilon$. Sinks could not be studied down to as low $\varepsilon$ as sources because at low $\varepsilon$ sinks tended to move quickly to the ends of the apparatus, where they were destroyed.

Over the range covered by our data, there is no systematic variation of the sink width, although of course we cannot rule out an increase in width at lower $\varepsilon$. Sinks are narrower than sources: the mean sink width is about 60% of that of the sources, although the different $\varepsilon$-dependence of the wave amplitudes near the defects makes a direct comparison difficult. The variability of the width, as indicated by the error bars, is also smaller for sinks both in absolute terms and relative to the mean width, and does not change significantly with $\varepsilon$. The decay lengths of the left- and right-going waves are also constant over the range of $\varepsilon$ studied. As was the case for the sources, the length scale of the left-going wave is larger than that of the right-going wave. The fact that the left-going wave is on the right-hand end of the apparatus in this case, and on the left-hand end for the sources, appears to rule out experimental imperfections as the reason for this difference.

The amplitude and wave number of the fingering pattern vary with both position and time near source and sink defects. For example, Fig. 3 shows that the amplitude of the right-going wave near the source develops substantial peaks that then move away from the defect. This variation in amplitude is mirrored in the local wave number of the pattern—a comparison of Figs. 3 and 4 shows that the regions of high amplitude near the source correspond to regions of high $k$. These
regions move with a speed equal to the speed of the fingers in the basic pattern. Although the signature of this behavior in the space–time plot is not obvious, these regions correspond to small groups of fingers emitted from the source with higher than average $\lambda$ and $k$. As they propagate away from the source, their amplitude and wave number both decrease. Similar behavior has been noted in this system previously [8].

Substantial localized depressions in the pattern amplitude are also seen. Fig. 11 shows the travelling-wave amplitudes near a source from which several such depressions are emitted; again they propagate with the speed of the underlying finger pattern. These depressions may be related to the “holes” discussed in the literature [17,23,24]. The depressions in amplitude are accompanied by corresponding decreases in $k$, that is, by a dilation of the finger pattern, and are associated with fingers that are noticeably slightly further apart than in the bulk of the pattern, as can be seen in the corresponding space–time image (Fig. 12). The travelling-wave amplitude in the depressions can be less than half that of the rest of the pattern, while the decrease in wave number is on the order of 20%. As in Fig. 11, the depressions typically die out after propagating some distance from the source.

It can be seen in Fig. 11 that emission of these depressions causes the position of the defect to change—the point at which $A_L$ and $A_R$ are equal drifts to the right as the amplitude of the right-going wave decreases, then back to the left as it rises again. In some cases, this process can make it difficult for our software to track the position of the defect. The space–time image (Fig. 12) also shows the defect moving about, although here the correspondence between the position of the defect found by our software and that estimated by eye is not always exact. Similar depressions are also seen in patterns dominated by sinks, in which case they are generated at the ends of the apparatus and propagate...
Fig. 8. A space–time image of a sink defect at $\varepsilon = 0.64$. The field of view is 136 mm, and the image covers 20 s in time. The black line running roughly vertically is the location of the defect.

in towards the sink. In most cases the depressions die out before reaching the sink, but in cases when they are absorbed by the sink the defect again changes position fairly suddenly as a result.

Slower-moving periodic traveling modulations of both $A$ and $k$ can also be seen in the images presented above. In Figs. 3 and 4, the modulations propagate away from a source, while in Fig. 9 they travel inwards towards a sink. These modulations may be related to the “modulated amplitude waves” found in numerical studies of the CCGLE\cite{17,25–27}. We observed these modulations on both sides of either kind of defect; in all cases the variations in $k$ and $A$ were in phase, that is, an increase in $k$ was accompanied by an increase in $A$. The amplitudes of the modulations observed were typically of order 10% in both $k$ and $A$, as in Fig. 5. Emission or absorption of these modulations by sources or sinks respectively can again result in jumps or other changes in the defect position, and can make our computerized tracking of the defects difficult, for the same reasons noted above. We measured the characteristics of these modulations from the $k(x, t)$ data, in which they were more clearly defined.

The traveling speed $v_m$ of the modulations is shown in Fig. 13(a). Near sinks, the modulations were not seen below $\varepsilon \approx 0.2$, and at $\varepsilon = 0.22$ they were intermittent and, when present, essentially stationary. Above $\varepsilon \approx 0.25$, however, they were always present. $v_m$ jumped from zero up to a value of approximately 4 mm/s, roughly a factor of six slower than the speed of the propagating fingers that make up the pattern. It then increased weakly with $\varepsilon$. Near sources, on the other hand, traveling modulations were present down to the lowest value of $\varepsilon$ studied, and their speed again increased slowly with $\varepsilon$. For $\varepsilon$ high enough that the modulations existed near both types of defects, their
Fig. 9. A space–time map of the amplitude of the left- and right-moving traveling into the sink of Fig. 8. As above, red indicates that $A_R$ dominates and blue that $A_L$ dominates. The black line running from top to bottom is the location of the defect.

Fig. 10. The amplitudes of the left-moving (dashed line) and right-moving (dotted line) waves traveling into the sink of Figs. 8 and 9 at the time indicated by the horizontal lines in those figures. The dotted line is a plot of $k$ at the same time.

The speed was the same in both cases. The spatial wavelength $\lambda_m$ of the modulations is shown in Fig. 13(b). Again there is no significant difference between modulations in patterns dominated by a source or by a sink. The wavelength shows a tendency to increase as $\epsilon$ decreases. Fig. 13(c) shows the temporal frequency $\omega_m$ of the modulations, determined from $\omega_m = 2\pi v_m/\lambda_m$; the frequency decreases steadily as $\epsilon$ is decreased towards the bifurcation.

4. Discussion

The bifurcation to the traveling finger state in this system is a subcritical parity breaking transition. On the other hand, existing CCGLE models of source and sink behavior are expected to apply near a supercritical Hopf bifurcation. Thus a direct connection with existing theory is less strong in our case than in some of the other experimental systems that have been studied.
Fig. 11. The amplitudes of the two travelling waves near a source at $\varepsilon = 0.46$. The field of view is 132 mm and the time covered is 12.5 s. Several localized depressions in amplitude are emitted from the sink, accompanied by changes in the position of the source.

Nonetheless many of our results are in at least qualitative agreement with behavior observed in the CCGLE simulations. The width of source defects in this system tends to increase as $\varepsilon$ decreases towards the onset of the travelling finger state. While our data do not extend to very small $\varepsilon$, they are consistent with a width that increases linearly with $\varepsilon^{-1}$. This is in accord with the predictions of the CCGLE model, and a similar divergence has been reported in experimental systems with supercritical bifurcations [1,3]. We also see an increase in the variability of the source width, and an increased tendency of the sources to wander about the pattern, at the lowest value of $\varepsilon$. This is suggestive of a transition in the stability of the source at low $\varepsilon$. The fact the wandering of the source becomes extreme and their lifetimes short at low $\varepsilon$ to the extent that we were unable to obtain data for $\varepsilon$ less than about 0.09 is additional evidence for such a transition. We are unable to say whether this transition is related to a transition from convective to absolute instability [18–21], although the behavior of the perturbations to the pattern seen in Fig. 7 indicates that this is a possibility. The wandering of the defect seen at low $\varepsilon$ is to be distinguished from the much smaller erratic motion of the sources which we observe at higher values of $\varepsilon$, which is associated with the emission of localized depressions or periodic modulations in the wave amplitude.

The sinks in our system are substantially narrower than the sources. Over the range of $\varepsilon$ covered by our data, neither the width of sink defects, nor the variability in that width, change significantly. Similar behavior has been observed in other systems [3,6]. While, in contrast, the CCGLE model predicts a divergence in sink width as $\varepsilon$ approaches zero.

The localized depressions in wave amplitude we observe may be related to the homoclinic holes found in solutions of the CCGLE, although it is worth emphasizing that we see localized enhancements of the amplitude as well. The depressions seen here are not
stable, since their amplitude decreases as they move away from the source. They move with the speed of the fingers in the underlying finger pattern, and the decrease in amplitude is accompanied by a decrease in $k$. The holes discussed by van Hecke [17, 23, 24] are also linearly unstable. In the solutions he describes the depression in $A$ is coupled with an increase in $k$, in contrast to what we observe, although Pastur et al. [3] report seeing both “dilation” and “compression” holes, corresponding to a decrease and an increase in wave number, respectively. Within our abilities to measure it, the wave number is equal on the two sides of the depressions, as found for the numerical homocrons. Nozaki–Bekki holes, in contrast, connect regions of different wave number [22]. Since our depressions move at the finger speed, they neither emit nor absorb waves, while the homocrons observed numerically emit waves on one side and absorb them on the other.

Similarly the traveling modulations in $A$ and $k$ that we observe may be related to the modulated amplitude waves described theoretically [17, 25–27]. These modulated amplitude waves appear when the underlying traveling-wave solution of the CCGLE is linearly unstable. While they are usually linearly unstable in the numerical work, in a certain range of parameters they appear via a subcritical bifurcation and can be stable. The modulations we observe appear to be stable—in their range of existence they are always seen to be regular and periodic, and no splittings or defects in the pattern of modulations were observed. Our data for modulations in patterns containing a sink suggest that they do in fact appear via a subcritical bifurcation around $\varepsilon \approx 0.21$, since their velocity jumps discontinuously from zero to a finite value there. For patterns with a source defect, however, the modulations are always present. This suggests that the presence of the...
5. Conclusions

We have studied the behavior of source and sink defects in traveling finger patterns in the printer’s instability, and of coherent structures seen in the traveling-wave amplitude and wave number. Many of our results are similar to those found in numerical simulations of coupled complex Ginzburg–Landau equations and in experimental work on other systems. In particular, we find that the width of source defects increases as $\varepsilon$ is decreased towards the onset of the traveling state, and the variability in their width increases at small values of $\varepsilon$. Our results are suggestive of a change in the stability of the sources at low $\varepsilon$, as found numerically. We see no such changes in the behavior of sink defects. Localized variations in the wave amplitude and wave number which propagate away from the sources with the same speed as the underlying finger pattern may be related to homoclinic holes found in numerical solutions of the CCGLE, and periodic traveling modulations in amplitude and wave number are reminiscent of the modulated amplitude waves described in the numerical work. The presence of a source destabilizes the finger pattern with respect to these modulations, while a sink stabilizes it.

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References


