Comparing limit laws for functions $y = f(x)$ and $z = f(x, y)$

$\lim_{x \to a} f(x) = L$:  
$\lim_{(x,y) \to (a,b)} f(x, y) = L$:  

| Sum, difference, product, quotient rules (provided separate limits exist and denominator limit is not zero). |
| Continuity (plug-in limits) for most reasonable functions in most places. |

If $\lim_{x \to a} f(x) = L$ and $w = g(y)$ is continuous at $y = L$, then

$\lim_{x \to a} g(f(x)) = g(L) = g(\lim_{x \to a} f(x))$

If $z = f(x, y)$ and $w = g(z)$ are both continuous, then so is $w = g(f(x, y))$.

Squeeze Theorem

One-sided limits

If either one-sided limit DNE or if the two one-sided limits are unequal, then $\lim_{x \to a} f(x)$ DNE.

If all path limits exist and equal $L$, then $\lim_{(x,y) \to (a,b)} f(x, y) = L$ (a USEFUL fact).

L’Hôpital’s Rule for some limits.

Path limits

If any path limit DNE or if any two path limits are unequal, then $\lim_{(x,y) \to (a,b)} f(x, y)$ DNE.

It is IMPOSSIBLE to use path limits to prove that $\lim_{(x,y) \to (a,b)} f(x, y)$ exists.

There is NO L’Hôpital’s Rule.