Most of combinatorics deals with enumeration or counting questions; that is, questions that begin “In how many ways can...?” Many such problems have been extensively studied, both because they are intrinsically interesting and because they arise in other areas of mathematics (as well as in other disciplines\(^1\)). The discoveries people have made while attempting to deal with such questions range from isolated tricks through techniques that can handle a handful of related questions to general theories and methods that are quite broad in scope. (You will see some of each of these in this course.) New discoveries have sometimes then generated new questions of their own, so that some areas of combinatorics (for example, the theory of Möbius inversion) have moved far beyond the concrete counting problems that started them off. There are also examples of counting problems that have so far defeated all attempts to solve them satisfactorily. An interesting example is that of the Stirling numbers. There are two kinds of these, and each kind satisfies a recurrence relation much like the recurrence for binomial coefficients,

\[
\begin{align*}
\binom{n}{k} &= \binom{n-1}{k} + \binom{n-1}{k-1}, \quad n \geq 1; \\
\end{align*}
\]

however there does not seem to be a formula for Stirling numbers analogous to the formula

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}.
\]

We will also spend some time on graph theory, which is very combinatorial in flavor. Graphs give rise to many interesting enumeration questions, and sometimes a problem that does not seem at all graphical will be solved by translating it into graph theory terms.

**Objectives.** By the end of the semester, the successful student

→ will have gained familiarity with basic combinatorial tools, such as binomial coefficients and Stirling numbers;

→ will be able to apply specific techniques (such as the Principle of Inclusion-Exclusion and Möbius Inversion) to combinatorial problems, including graph-theoretical problems;

→ will have acquired familiarity with basic graph theory (e.g., the Handshaking Lemma, characterizing Eulerian graphs, basic facts about trees)

→ will be able to apply graph theory techniques to combinatorial problems;

→ will have acquired some facility with generating function techniques.

**Homework.** There will be frequent homework sets, each of which will carry a due date and a resub date (see below). The possible grades on most problems will be 0/4/7/9/10, where

\(^1\) Offhand, I can think of examples from chemistry, from biology, and from the theory of experimental design.
“10” means “completely correct,”
“9” means “almost completely correct,”
“7” means “you made progress,”
“4” means “you got started,” and
“0” means “you didn’t get started.”

Please turn in your homework on 8\frac{1}{2}” × 11” paper, leaving the right third of each sheet blank; this leaves room for my comments. Also, please preface each solution you hand in with a statement of the problem you are solving (including page number and problem number, when the problem comes from the text); this simple step will make your problem sets much more useful for studying and reviewing.

It is extremely important that you keep up with the homework, which is really the heart of the course. The only way to learn this material is by doing; you must put in a lot of time concentrating on the concepts and techniques that will be flying at you in order to master them. This includes time spent staring at hard problems and maybe getting nowhere; time spent like this is NOT wasted. You should probably plan to spend about 15 hours per week, preferably in several separate sessions, on this task. I strongly encourage you to work together on homework, and you may consult me or other members of the department. However, you must write the solution to each problem on your own; you must tell me with whom you worked on each problem (if anyone); and you must cite any help you receive from faculty members (including me).

Resubmission of problems. You may usually resubmit one solution for each problem. Any solution that earns a 7 or a 9 may be resubmitted, but occasionally I will not allow a resubmission of a solution that earns a 0 or a 4 (generally because the solution shows insufficient effort). If I won’t be allowing a resubmission, my comments will indicate this clearly, so you may take the absence of such a comment as implicit permission to resubmit.

There are three restrictions on resubmissions. First: Although I encourage you to work on homework together in groups, you may NOT discuss resubmissions with anyone EXCEPT me; I want this work to be solely yours. Second: If you wish to resubmit an eligible 4 or 0, you MUST discuss the problem with me first; I want to make sure you understand what corrections are needed. Third: Each resubmission must be accompanied by the original submission.

Tests and Grading. There will be a take-home midterm and a take-home final. I will also average your homework grades, so that at the end of the semester, you will have three scores (midterm grade, grade in final exam, homework average). In computing your final average I will weight these scores as follows:

- midterm: 20%
- final exam: 30%
- homework average: 50%