Eighteen Short Proofs

1. Let \( x, y \in \mathbb{R} \). If \( xy > 0 \) and \( x + y > 0 \), then \( x > 0 \) and \( y > 0 \).
2. There are no integers \( m \) and \( n \) such that \( 9n - 6 = 27m \).
3. Let \( A \) and \( B \) be sets. If \( B \subseteq A \), then \( A \cup B = A \).
4. Let \( R \) be the relation from \( \mathbb{R} \) to \( \mathbb{R} \times \mathbb{R} \) defined by \( xR(y, z) \) if \( x = y \). Then
   (a) \( R \) is not a function.
   (b) \( R^{-1} \) is a function.
5. There exists no largest negative real number.
6. The function \( f: \mathbb{Z} \to \mathbb{N} \) given by \( f(n) = |n| + 1 \) is not a bijection.
7. The open interval \((3, 6)\) is uncountable. (You may assume that \((0, 1)\) is uncountable.)
8. The sum of the smallest \( n \) odd positive numbers equals \( n^2 \).
9. Let \( x, y \in \mathbb{Z} \). Then \( xy \) is even if and only if \( x \) is even or \( y \) is even.
10. Let \( R \) be the relation defined on \( \mathbb{Z} \times \mathbb{Z} \) by \((a, b)R(c, d)\) if \( a = c \). Then \( R \) is an equivalence relation. (What are its equivalence classes?)
11. Let \( f: A \to A \) be a function. Then \( f \circ i_A = i_A \circ f = f \), where \( i_A \) is the identity function on \( A \).
12. Let \( x, y, n \in \mathbb{Z} \) and let \( n \geq 2 \). If \( x \equiv y \pmod{n} \), then for all \( k \in \mathbb{N} \), \( kx \equiv ky \pmod{nk} \).
13. Let \( A \) and \( B \) be sets. If \( A \) and \( B \) are denumerable, then \( A \) and \( B \) are numerically equivalent.
14. For all \( n \in \mathbb{N} \), \( 6 \mid (7^n - 1) \).
15. There exists no irrational number whose square root is rational.
16. Every finite nonempty set of real numbers has a largest element. (Hint: Use induction on the size of the set.)
17. Let \( A \) be a nonempty set and let \( f: A \to \mathcal{P}(A) \) be the function \( f(x) = \{x\} \). Then \( f \) is an injection but not a surjection.
18. The function \( f: \mathbb{R} - \{1\} \to \mathbb{R} - \{2\} \) defined by \( f(x) = \frac{2x - 1}{x - 1} \) is a bijection.