The Mathematics of Poetry
Multicultural Mathematics
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The mathematical field of **Combinatorics** includes the mathematics of combinations and permutations. Combinatorics starts with questions of how many ways there are to do or make something. Some early studies in Combinatorics were done by Ancient Indian scholars studying poetry!

Hemacandra. The Jain writer Ācārya Hemacandra (c. 1150 AD) studied the rhythms of Sanskrit poetry. Syllables in Sanskrit are either long or short. Long syllables have twice the length of short syllables. The question he asked is *How many rhythm patterns with a given total length can be formed from short and long syllables?*

For example, how many patterns have the length of five short syllables (i.e. five "beats")?

There are eight:

SSSSS, SSSL, SSLS, SLSS, LSSS, SLL, LSL, LLS

As rhythm patterns, these are

`xxxxx, xxxx., xxx.x, xx.xx, x.xxx, xx.x., x.xx., x.x.x`

To investigate the general rule, we can make a chart:

Patterns	Length	Number
S	1	1
SS, L	2	2
	3	
	4	
	5	8
	6	

It turns out that the sequence of numbers of patterns is called the Fibonacci sequence, after the Italian mathematician Fibonacci, whose work was published 70 years *after* Hemacandra's. The numbers in the sequence are called Fibonacci numbers. In your own words, give the rule for finding the next number in the Fibonacci sequence.

Returning to our musical question, the answer is that *the number of rhythm patterns with length n is the sum of the number of patterns of length $n-1$ and the patterns of length $n-2$.*

Why? Write out the eight patterns of length five beats in a special way. First list the ones that start with a L—there are three of these:

Now list the five patterns that start with a S:

Write out the three patterns of length 3:

and the five of length 4:

Explain how the patterns of length 3, 4, and 5 are related:

Pingala. Another question inspired by the study of rhythm is: how many patterns can be formed from a given number of syllables? For example, how many patterns consist of three syllables? The syllables can be all L's, all S's, or some mixture of L's and S's. The answer is eight; see if you can discover how I grouped them.

SSS	SSL	SLL	LLL
	SLS	LSL	
	LSS	LLS	

Pingala's *Chandahsutra* (c. 200 B.C.) enumerated the possible poetic meters of a fixed number of syllables. Syllables are short (1 beat) or long (2 beats).

He classified 16 different meters of four syllables in the following way:

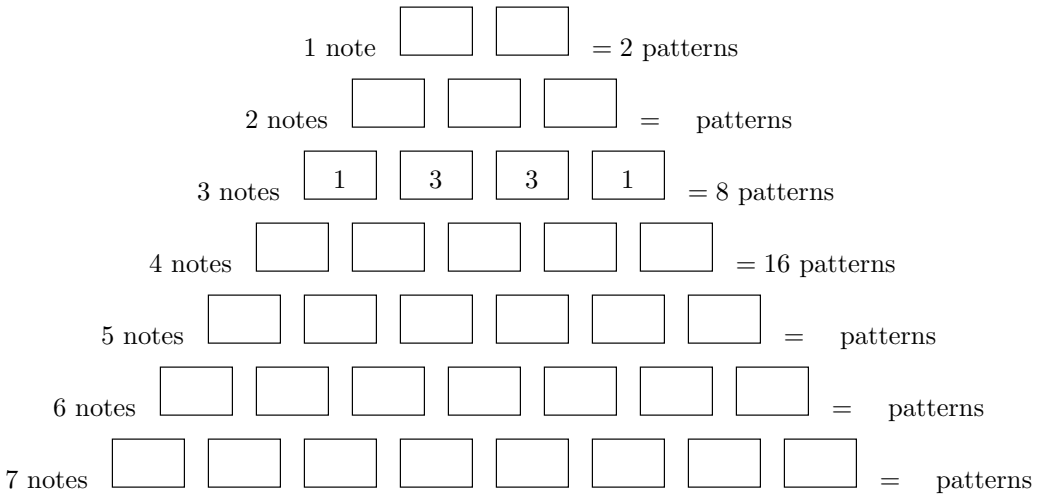
1	meter of four short syllables
4	meters of three shorts and a long
6	meters of two shorts and two longs
4	meters of one short and three longs
1	meter of four longs

Now find the number of patterns consisting of two syllables, grouped by length:

Let's make a chart of the results and look for a pattern:

Number of syllables	Number of patterns	Grouping of patterns
1		
2	4	
3	8	$1 + 3 + 3 + 1$
4	16	
5		$1 + 5 + 10 + 10 + 5 + 1$
6		$1 + 6 + 15 + 20 + 15 + 6 + 1$

We see that the answer to our original question is that *the number of patterns composed of n syllables is* 2^n. In addition, there's something really interesting going on with the groupings of patterns. Fill in the following triangle with your values for the groupings:



This is called Pascal's Triangle!

This observation was first made by the Indian writer Pingala (c. 200 B.C.), who lived *eighteen* centuries before Pascal! It also appears in the commentary on this work by Halayudha (c. 10th century).