Let $M = n\ell$, and let $D_M$ act on $S^n_\ell$. The number of orbits is

\[
\begin{align*}
\frac{1}{2M} \left[ \sum_{d | M, \gcd(d,\ell) = 1} \phi(d) + \sum_{d | n, \gcd(d,\ell) = 1} \phi(d)(\ell + 1)^{n/d} + 2M(\ell + 1)^{n-1} \right], & \quad \text{if } n \text{ is odd;}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2M} \left[ \sum_{d | M, \gcd(d,\ell) = 1} \phi(d) + \sum_{d | n, \gcd(d,\ell) = 1} \phi(d)(\ell + 1)^{n/d} + M \left( \frac{\ell + 5}{2} \right) (\ell + 1)^{n-2} \right], & \quad \text{if } n \text{ is even and } \ell \text{ is odd; and}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2M} \left[ \sum_{d | M, \gcd(d,\ell) = 1} \phi(d) + \sum_{d | n, \gcd(d,\ell) = 1} \phi(d)(\ell + 1)^{n/d} + M \left( \frac{\ell + 4}{2} \right) (\ell + 1)^{n-2} \right], & \quad \text{if } n \text{ is even and } \ell \text{ is even.}
\end{align*}
\]

The new ingredients in the formulas are the sum of $\text{fix}(\beta)$, summed over all reflections $\beta$. Here are the expressions for $\text{fix}(\beta)$ that I found. In the case $\ell$ and $n$ both odd, all $M$ reflections are essentially $\beta(x) = -x \pmod{M}$. In the other three cases, $M/2$ are of that form, and the other $M/2$ are of the form $\beta(x) = 1 - x \pmod{M}$.

<table>
<thead>
<tr>
<th>parity of $n$</th>
<th>parity of $\ell$</th>
<th>$\beta(x) = -x$</th>
<th>$\beta(x) = 1 - x$</th>
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</thead>
<tbody>
<tr>
<td>odd</td>
<td>odd</td>
<td>$\text{fix}(\beta) = 2(\ell + 1)^{n-1}$</td>
<td>$\text{fix}(\beta) = 2(\ell + 1)^{n-1}$</td>
</tr>
<tr>
<td>odd</td>
<td>even</td>
<td>$\text{fix}(\beta) = 3(\ell + 1)^{n-1}$</td>
<td>$\text{fix}(\beta) = (\ell + 1)^{n-1}$</td>
</tr>
<tr>
<td>even</td>
<td>odd</td>
<td>$\text{fix}(\beta) + 4(\ell + 1)^{n-2}$</td>
<td>$\text{fix}(\beta) + (\ell + 1)^{\frac{n-2}{2}}$</td>
</tr>
<tr>
<td>even</td>
<td>even</td>
<td>$\text{fix}(\beta) = 3(\ell + 1)^{n-2}$</td>
<td>$\text{fix}(\beta) = (\ell + 1)^{\frac{n-2}{2}}$</td>
</tr>
</tbody>
</table>