Music and Periodicity

According to the French composer Edgar Varese, “Music is organized sound.” Let’s set aside, for the moment, a discussion of whether or not we agree with Varese, and look at some examples of “organization” in musical sound. We’ll start by looking in detail at some sound clips.

Here’s a portion of the soundfile from the ZZ Top clip. The numbers on top indicate seconds. You can see that the total length of the selected portion is 4.835 seconds at the bottom of the window.

Here’s a portion of the soundfile from the cello sound. You can see that I’ve zoomed in quite a bit compared to the ZZ Top file.

Each of these sounds is roughly periodic, meaning that it repeats the same pattern at regular time intervals.

The fundamental period of anything periodic is the length (in time) of the shortest pattern that can be repeated to form the overall pattern. A fundamental cycle of a periodic pattern is any portion of the overall pattern whose length equals the fundamental period. For example, the fundamental period of the ZZ Top soundfile looks to be about 0.5 seconds. (Of course, the file is only approximately periodic—the drum beat repeats, but other sounds don’t.)

Here’s an example of a sound file for a sawtooth wave. In this case, the sound is generated by a computer, so it’s exactly periodic, while the examples we saw before were only approximately periodic.
I can see that the length of the clip shown is exactly 0.1 second. However, a shorter pattern can be repeated to make the entire clip, something like

this  or this  or this  or even this

The first three patterns are examples of fundamental cycles, because their length is the shortest possible. The fourth pattern does repeat to form the entire soundfile, but it’s not a fundamental cycle because it can be subdivided into shorter patterns that repeat.

Let’s figure out the fundamental period of this sound. You can get usually a more accurate measurement by finding the length of ten fundamental cycles, then dividing that number by ten. In this case, the length of ten fundamental cycles is exactly 0.1 second, so each fundamental cycle lasts 0.1/10 = 0.01 second. This is the fundamental period.

EXERCISE 1. Find the fundamental period of the ZZ Top drum beat and of the vibrations of the cello sound. Use correct units in your answer.

ZZ Top: ____________  Cello: ____________

EXERCISE 2. Sketch one fundamental cycle of the cello sound. Then, make a sketch that is not a fundamental cycle but can be repeated to form the overall pattern.

Now we’re going to check our work by making soundfiles that repeat with the same fundamental period. The first thing I need to know is the frequency of the repetition, meaning the number of fundamental cycles per unit of time (usually per second or per minute). I calculated that the fundamental period of the sawtooth wave is 0.01 second. That is, the vibration is

0.01 seconds per cycle,  or  \(0.01\text{ seconds per cycle}\).
However, frequency is measured in cycles per second, or \( \text{cycles} \over \text{second} \). Dimensional analysis tells us to take the reciprocal to find the frequency:

\[
\frac{0.01 \text{ seconds}}{\text{cycle}} \text{ corresponds to } \frac{1}{0.01 \text{ second}} = 100 \text{ cycles per second.}
\]

“cycles per second” is normally called *Hertz* (abbreviated Hz), so we say that the frequency is 100 Hz.

**Exercise 3.** Find the frequency of the ZZ Top and cello sounds, measured in Hertz.

ZZ Top: \[\_\_\_\_\_\_\_\_\_\_\]  Cello: \[\_\_\_\_\_\_\_\_\_\]

Of course, there are basic differences between the cello sound and the ZZ Top music. We hear a fast vibration as a smooth *pitch*. The limits of human perception of pitch are roughly 20 Hz to 20,000 Hz (also written as 20 kHz, for kilohertz). We hear a slower repeated sound as a *beat*, and we refer to its frequency as the *tempo* of the music. Tempo is usually measured in beats per minute (BPM) rather than Hertz. To convert from Hertz from BPM, use dimensional analysis:

\[
1 \text{ Hz} = \frac{\text{cycles}}{\text{second}} \times \frac{60 \text{ seconds}}{\text{minute}} = 60 \frac{\text{cycles}}{\text{minute}} = 60 \text{ BPM.}
\]

**Exercise 4.** Find the tempo of the ZZ Top example, measured in BPM.

*Making a dance mix.* Suppose you have two songs that don't have exactly the same tempo. Audacity is able to change the tempo of a song without changing its pitch (that is, without making it higher or lower). You could use Audacity to make a smooth transition between songs without changing the beat. (Some software does this automatically, but it’s good to understand how it works.) The idea is to open the songs as separate tracks, find the tempo of each, then use “Effect / Change Tempo” to make them have the same tempo. You can then drag the files forward or backward so the beats line up. To make the transition smooth, create a “crossfade” by fading one song out while fading the other in.
Answers to Exercises.

Exercise 1. In each file, exactly ten fundamental cycles are shown, so I find the length of ten fundamental cycles and divide by 10. The fundamental period of the ZZ Top drum beat is $4.835/10 = 0.4835$ seconds. The fundamental period of the cello sound is $0.102/10 = 0.0102$ seconds.

Exercise 2. Answers vary, but the length of a fundamental cycle should always be 0.0102 seconds. Here’s a possible answer:

Here’s a picture that is not a fundamental cycle, but can be repeated to get the entire file:

Any whole number of fundamental cycles (greater than 1) will work.

Exercise 3. For ZZ Top, 
\[
\frac{1}{0.4835 \text{ seconds/cycle}} = 2.068 \text{ cycles/second} = 2.068 \text{ Hz}.
\]

For the cello sound, 
\[
\frac{1}{0.0102 \text{ seconds/cycle}} = 98.04 \text{ cycles/second} = 98.04 \text{ Hz}.
\]

Exercise 4.
\[
2.068 \text{ Hz} = 2.068 \text{ cycles/second} \times \frac{60 \text{ seconds}}{\text{minute}} = (2.068 \times 60) \text{ cycles/minute} = 124.08 \text{ BPM}.
\]