Reviews


David Lewin (1933–2003) was one of the twentieth century’s most influential music theorists. He, more than anyone, is responsible for the flourishing of the mathematical branch of music theory in the late twentieth century and into the twenty-first. Twenty years after the publication of his seminal work *Generalized Musical Intervals and Transformations* (commonly known as *GMIT*) and fourteen years after the publication of *Musical Form and Transformation: Four Analytic Essays*, Oxford University Press offers timely new editions of both works. The 2007 edition of *GMIT* contains a previously unpublished thirteen-page preface written by Lewin and a foreword by Edward Gollin; otherwise, the body of the text has been preserved.¹ *Musical Form and Transformation* (*MFT*) has been reprinted in its original form. This essay cannot do justice to the vast number of ideas Lewin introduces and applies in *GMIT* and *MFT*. I hope to point the reader towards the aspects of his work I find the most powerful, engaging, and challenging.²

How should one evaluate Lewin’s work? The question is a surprisingly difficult one. Mathematical music theory is neither pure mathematics nor truly an applied science. Lewin’s goal is to develop abstract musical spaces that model his musical intuitions and to use these spaces in cogent analyses. On the one hand, applied mathematicians—economists, for example—sometimes appeal to intuition when proposing models but normally use empirical data to test their theories. Although Lewin occasionally refers to studies on music perception, he is not much concerned with testing his theories empirically. On the other hand, pure mathematicians and theoretical physicists explore abstract spaces; however, a physicist would hardly announce her or his intention to “model our intuitions about the shape of the universe.” The first two parts

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¹. This fidelity includes the pagination of the body text but not of the introduction. Lewin identified a few typographical errors that the new edition corrects.

². I thank Dmitri Tymoczko and Steven Rings for their helpful comments on earlier drafts of this review.
of this review consider two of Lewin’s most powerful constructions: generalized interval systems and transformational networks. In the final section, I address four questions in turn: (1) Is the mathematics correct? (2) Do generalized interval systems and transformations model our intuitions about abstract musical structures in a sufficient number of situations? (3) Are Lewin’s musical analyses persuasive? (4) Setting aside musical considerations, how does GMIT hold up as a math book?

Both books are well known and have been reviewed many times. As my own background is in mathematics, this review engages Lewin’s mathematics rather more than others reviews have done. I summarize some of the key mathematical concepts of GMIT in the first part of this review and discuss their implementation in the second and third parts. The nonmathematical reader may wish to skip to the second section (“Transformational analysis,” p. 212) and return to the beginning as needed. Likewise, I would encourage new readers of Lewin—especially those without a mathematical background—to begin by reading his musical analyses and then make inroads into the mathematical discussion.

Generalized interval systems and transformation groups

GMIT opens with a picture (Fig. 1) symbolizing the aim of the first four chapters: to draw an analogy between the historical concept of “interval” and the “characteristic gestures” one hears in actual music. We are to imagine that \( s \) and \( t \) are musical structures belonging to the same “family” (for example, two pitches, two pitch classes, two chords, or two series) and that \( i \) is some “directed measurement, distance, or motion” that proceeds from \( s \) to \( t \). Lewin’s description of \( i \) as an “interval” is metaphoric: Lewinnian intervals are far more than distances in pitch or frequency ratios. Rather, they model, among other things, transpositions between pitches, pitch classes, chords, and series; serial transformations; and rhythmic relationships.

A central idea of chapters 2–4 is that intervals naturally form a mathematical “group.” A group is a set of objects in which any two objects can be combined to produce a third object in the group (their “product”), subject to some restrictions.\(^3\) For example, the real numbers form a group, where the

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3. Formally, a group is a (possibly infinite) set \( G \) together with a product “\( \cdot \)” that combines any ordered pair \( (f, g) \) of the group’s elements to produce an element \( h \) that also belongs to the group (we write \( f \cdot g = h \)). A group must obey certain rules: (1) It must be closed, meaning that its product associates every pair of its elements to some other group element. (2) It has an identity, meaning a special element \( e \) such that \( g \cdot e = e \cdot g = g \) for every \( g \) in the group. (3) Every element \( g \) in the group has an inverse, or element \( g^{-1} \) such that \( g \cdot g^{-1} = g^{-1} \cdot g = e \). (4) The group is associative. \( f \cdot (g \cdot h) = (f \cdot g) \cdot h \) (“\( f \) times \( g \) times \( h \)” is unambiguous). The use of groups in music theory is most closely associated with Lewin and Milton Babbitt. However, groups—although
combination of two numbers is their sum. The numbers \{0, 1, 2, \ldots, 11\} constitute the group of “integers modulo 12” whose product is addition modulo 12, meaning that the product of \(x\) and \(y\) equals \(x + y\) if \(x + y\) is less than twelve and equals \(x + y - 12\) if \(x + y\) is at least twelve (this product is also called “clock arithmetic”). However, these examples are not close to representative. Group theory is a vast subject intersecting with virtually every field of pure mathematics and some fields of applied mathematics. The concept of “group” is powerful precisely because it is independent of any particular application: two groups appearing in different contexts are considered “the same” (isomorphic) if they share the same abstract structure.

Lewin defines a “generalized interval system” (GIS) to be a set of musical objects \((S)\), a set of “intervals” (IVLS) that form a group, and an “interval function” \((\text{int})\) that assigns a unique interval to each ordered pair of objects. The GIS construct can reveal similarities between different musical spaces: diatonic and chromatic scales \((GMIT, p. 17)\), pitch classes and beat classes \((GMIT, p. 23)\), and so on. Example 2.1.2 \((GMIT, p. 17)\) models our perception that directed intervals in pitch have size and add in the same way that numbers do: “two semitones up” followed by “five semitones down” equals “three semitones down”:

\[\text{Example 2.1.2. The musical space is the gamut of chromatic pitches in twelve-tone equal temperament (12-tet). The interval between two pitches } s \text{ and } t \text{ is}\]


4. The real numbers lie on a continuum (the “number line”) and the whole numbers constitute the set of integers.

5. The connection between pitch class and beat class was one of Babbitt’s chief motivations for introducing groups in music theory.

6. In the interests of streamlining this discussion, I have paraphrased this example and the ones following rather than quoting directly from Lewin’s book.
the number of upward chromatic steps from \( s \) to \( t \). So the interval from C4 to D4 is 2 and the interval from C4 to G3 is –5. Intervals constitute a group (the integers) and combine by addition.

Example 2.1.2 requires a certain suspension of disbelief: in order for its intervals to form a group, it must contain arbitrarily large intervals. Lewin comments: “Application of this idea may require enlarging practical families of musical elements, to become larger formal spaces that are theoretically conceivable while musically impractical” (GMIT, p. 27). Example 2.1.3 (p. 17) models the pitch-class circle:

Example 2.1.3. The musical space is the twelve pitch classes in 12-tet. The interval between two pitch classes \( s \) and \( t \) is the number of clockwise chromatic steps on the pitch-class circle from \( s \) to \( t \). For example, the interval from C to D is 2 and the interval from C to G is 7. Intervals constitute a group (the integers modulo 12) and combine by addition modulo 12.

Lewin’s assertion that intervals model directed distance, measurement, or motion seems entirely fitting in these situations. In general, however, mathematical groups have structure that cannot be modeled by arithmetical operations, and so the words “distance” and “measurement” become problematic. Lewin presents the following example—the TPERM GIS—in the new preface to GMIT:

The TPERM GIS. LAGEN is the set consisting of all permutations of tunes A, B, and C (that is, \{<A–B–C>, <A–C–B>, <B–A–C>, <B–C–A>, <C–A–B>, <C–B–A>\}) and TPERMS is the group of six actions that rearrange the tunes in one permutation to produce another. The “interval” from one permutation to another is the unique action that inputs the first permutation and outputs the second. For example, the interval between <A–B–C> and <A–C–B> is “exchange the second two tunes.” (GMIT, p. xviii)

The difference between this example and the previous ones is striking. It is difficult to imagine the elements of the TPERM GIS as “intervals” in any traditional sense (although it is not so difficult to check that the definition of a

7. Note that undirected intervals do not form a GIS: “a perfect fifth” has no inverse.

8. Lewin attributes this GIS to Daniel Harrison (“Some Group Properties of Triple Counterpoint and Their Influence on Compositions by J. S. Bach,” Journal of Music Theory 32 [1988]: 23–49). Unlike the others we have considered, its interval group is noncommutative—that is, order matters when computing products. Lewin was particularly interested in exploring such groups.

Groups of actions that rearrange sequences of objects are called permutation groups. A fundamental theorem of group theory holds that every finite group is equivalent to some subgroup of a permutation group. Music theorists (and, indeed, mathematicians) have long explored questions involving permutations of various musical objects (see Marin Mersenne’s Harmonie Universelle [1636]). However, they have typically asked questions such as “What are the possible arrangements of A, B, and C?” or “What is the relationship between ABC and ACB?” Transformational theory asks, rather, “What ‘characteristic gesture’ acts on ABC to produce ACB?”
GIS is satisfied). There is no natural way to measure permutations—of course, we can assign sizes to them, but without additional information about what we are trying to model, that assignment is arbitrary. Lewin does not mention “distance” or “measurement” in connection with the TPERM GIS.

What, then, of Lewin’s stated intention to use intervals in a GIS to model distance, measurement, or motion? In fact, he ignores distance in connection with quite a few examples in GMIT, including TPERM. Moreover, while it is true that measuring 12-tet pitch-class intervals in semitones (as in ex. 2.1.3) models a different notion of distance than measuring intervals in perfect fifths does (as on page 22 of GMIT), these notions are not encoded in the structure of the corresponding GISes, but rather conveyed by the names of their intervals. The most noticeable difference between the two is that the former extends naturally to the continuous pitch-class circle, while the latter does not. However, the formal definition of a GIS sets aside such considerations. I conclude that GISes can express notions about distance, but are not forced to do so.

Chapters 7–9 of GMIT consider transformations—“actions” that one performs in order to move from one musical object to another. Formally, transformations belong to mathematical groups or semigroups “acting” on a set of

9. This question has been the subject of considerable debate. Both “distance” and “measurement” have precise mathematical meanings. My reading, based on a passage in chapter 4, is that Lewin does not mean them in a strict sense: the discussion of Theorem 4.1.5 (pp. 77–80) reveals that he considers a family of GISes “‘essentially’ the same” in the sense that (1) they concern the same set of musical objects and (2) their respective interval functions define the same equivalence relations on the set of ordered pairs of these objects (this forces their interval groups to be isomorphic). Therefore, I take it that their respective notions of distance are “inessential” information. According to these criteria, measuring distance on the pitch class circle by semitones or measuring it by perfect fifths produces “‘essentially’ the same” GIS. Both systems consider the intervals from C to G and from A to E to be the same and combine the two intervals to produce the interval from C to D. Of course, if we want to do anything musically useful, we should not assign “sizes” to intervals in an arbitrary way: a “size” is not just a “name.” Edward Henry Gollin (“Representations of Space and Conceptions of Distance in Transformational Music Theories” [PhD diss., Harvard University, 2000]) develops an appropriate notion of distance using a “word length metric,” and Tymoczko (“Generalizing Musical Intervals,” preprint, 2008) proposes a “Lewinnian interval system”—a GIS together with a distance function—that subsumes Gollin’s construction.

My reading of Theorem 4.1.5 agrees with Dan Tudor Vuza’s definition of “GIS equivalence” (“Some Mathematical Aspects of David Lewin’s Book: Generalized Musical Intervals and Transformations,” Perspectives of New Music 26 [1988]: 258–87). In contrast, Oren Kolman (“Transfer Principles for Generalized Interval Systems,” Perspectives of New Music 42 [2004]: 150–91) defines “GIS isomorphism” to mean isomorphism of interval groups, but misses an opportunity to distinguish the special case where two GISes on the same set are equivalent up to the labeling of their intervals, as in Theorem 4.1.5. STRANS1 and STRANS2 in Appendix 2 of GMIT are “isomorphic” by Kolman’s definition, but not “‘essentially’ the same” in the sense of Theorem 4.1.5 or Vuza’s “equivalent.” In an attempt to dissociate the GIS structure from any notion of distance, Tymoczko showed that every GIS has a “canonical unlabeled interval group” that names intervals only by “what they do” (“Lewin, Intervals, and Transformations: A Comment on Hook,” Music Theory Spectrum 30 [2008]: 164–68). It is not difficult to show that two GISes with the same set S of musical objects share the same canonical unlabeled interval group if and only if they are “‘essentially’ the same.”
Performing an action (a member of the group or semigroup) on any object in the set returns another object in the set. Actions are combined by “composition,” or successive application (“do $a$ then do $b$”). Transformations that can be undone by some other transformation are called “operations.” For example, transpositions are transformations acting on pitches, chords, or melodies, as in the following example, which Lewin uses in many contexts (cf. GMIT, pp. 165–69; MFT, chap. 4):

Transposition group 1. The transpositions $T_i$, where $i$ belongs to the integers modulo 12, form a group of “pitch-class transpositions” acting on the 12-tet pitch classes. If $X$ is the pitch class labeled by the number $x$, then $T_i(X)$ is the pitch class labeled by $i + x$ modulo 12. The product of $T_i$ and $T_j$ is $T_k$, where $k$ is the sum of $i$ and $j$ modulo 12.

Transposition group 1 naturally associates with the GIS of pitch-class intervals (ex. 2.1.3). In fact, the identification of transpositions with intervals in pitch or pitch class is one of Lewin’s central constructions. Although the connection between intervals in the fundamental bass and harmonic transformations is well established historically, his model is more general. For example, a succession of transpositions of local tonics in act 1 of Wagner’s Parsifal elaborates on the Zauber motive $A\flat$–$E\flat$–$E\flat$ that is to appear in act 2 (GMIT, pp. 161–64). Lewin draws a similar example from Schoenberg’s Piano Piece Op. 19, no. 6 (GMIT, pp. 159–60).

Is there any mathematical difference, then, between intervals and transformations? Why does Lewin introduce them as distinct concepts? In one of the most quoted passages of GMIT, he describes the contrast between the intervallic, “Cartesian,” view and the transformational one:

We tend to imagine ourselves in the position of observers when we theorize about musical space; the space is “out there,” away from our dancing bodies or singing voices. “The interval from $s$ to $t$” is thereby conceived as modeling a relation of extension, observed in that space external to ourselves; we “see” it out there just as we see distances between holes in a flute, or points along a stretched string. . . . In contrast, the transformational attitude is much less Cartesian. Given locations $s$ and $t$ in our space, this attitude does not ask for

10. Semigroups are closed and their products are associative. However, semigroup elements need not have inverses and semigroups need not have an identity element. Every group is a semigroup. The rules of semigroup formation ensure that (1) the process of applying any two actions in succession (“do $a$ then $b$”) is itself an action included in the group, and (2) the meaning of the action “do $a$ then $b$ then $c$” is unambiguous: following the action “do $a$ then $b$” by action $c$ produces the same result as following action $a$ by the action “do $b$ then $c$.” If the semigroup is a group, then (3) there is at least one member of the group that “does nothing” to every element in the set, and (4) every action can be undone by at least one other action.

11. That is, $i$ is the interval between pitch classes $s$ and $t$ if and only if $t$ is the “$i$-transpose” of $s$ ($t = T_i(s) = i + s$ modulo 12). We could recast Transposition group 1 as a GIS whose objects are pitch classes and whose intervals are transpositions; this GIS is identical to ex. 2.1.3 modulo the renaming of intervals by the recipe “$i$ corresponds to $T_i$.”
some observed measure of extension between reified “points”; rather it asks: “If I am at \( s \) and wish to get to \( t \), what characteristic gesture (e.g. member of [a group acting on the space]) should I perform in order to arrive there?” (p. 159)

Although Lewin comments in the introduction to GMIT (p. xxxi) that “interval-as-extension” and “transposition-as-characteristic-motion-through-space” are “various aspects of the same basic phenomenon,” he uses them in slightly different situations. My reading is that the “intervallic” perspective “observes” the musical space and intuits which relationships between objects are equivalent “\( A \) is to \( B \) as \( C \) is to \( D \)” (or, as Lewin comments in his Stockhausen analysis “\( P8 \) is to \( p8 \) as \( P \) is to \( p \)” [MFT, p. 26]). This intuition may be founded in some notion of distance, but, as in the TPERM GIS, it need not be. The transformational attitude, on the other hand, “knows” which characteristic gestures may be performed. When every ordered pair of objects is connected by a unique transformation, the two perspectives are formally equivalent: every GIS’s interval group acts “simply transitively” on its set of objects, meaning that there is exactly one directed interval (“action”) between any two objects. However, in the transformational situation, there may be multiple actions connecting one object to another or even none at all. Only “certain structures of transformations” correspond to GISes (GMIT, p. xxxi).12 Lewin refers to transformations that cannot belong to a GIS as “non-intervallic,” although he never defines the term explicitly. Transposition group 2, though not presented by Lewin, creates an instructive contrast with Transposition group 1. Its transformations are nonintervallic.13

**Transposition group 2.** The set of (arbitrarily large) 12-tet transpositions forms a group of “pitch transpositions” that acts on the set of 12-tet pitch classes. For example, \( T_{14}(C) = D \) and \( T_{-4}(C) = A_{\flat} \).

No GIS whose set is the 12-tet pitch classes can have intervals that run the gamut of integers. The obstruction is that the interval between two pitch classes does not correspond to a unique transposition (in other words, the group of 12-tet transpositions does not act simply transitively on the set of

12. In the quoted passage, “transformational” pertains to a “member of a simply transitive group”—that is to say, an operation that belongs to a simply transitive group. To replace “characteristic gesture” with either “characteristic gestures” or “admissible transformations” (as Lewin does in the next paragraph) would have been more consistent. If such a gesture does not exist in every situation, the action of the “admissible transformations” is deemed “not transitive.” If the admissible transformations belong to a group or semigroup that acts transitively but not simply transitively, there may be multiple transformations that do the trick.

pitch classes). For example, multiple transpositions input C and output E; $T_{16}$ would do as well as $T_4$.

The transformational and intervallic attitudes offer different possibilities for musical analysis. On the one hand, the intervallic perspective offers a reassuring certainty—there is exactly one interval between any two objects, so, within the closed system of a GIS, analysis seems more or less objective. This is particularly attractive in atonal music, where we may have fewer preconceived ideas about which transformations are “admissible.” On the other hand, the rich theory of tonal music offers multiple readings of the same passage; the flexibility of the transformational attitude is often more appropriate (for example, Wagner’s progression from G-flat major to B-flat minor in the Valhalla theme of *Das Rheingold*, scene 2, mm. 1–20, can be effected by two different triadic operations, one viewing B-flat minor as the submediant of G-flat major and the other employing the Riemannian *Leittonwechsel* operation [GMIT, p. 178]). In the following chapters, Lewin presents other examples of “non-intervallic” transformations in the works of Wagner, Webern, Beethoven, Mozart, Bartók, Prokofiev, and Debussy.  

**Transformational analysis**

The remaining chapters of *GMIT* propose a formal theory of transformational analysis and demonstrate its utility. A “transformation graph” consists of “nodes” that are normally given generic names ($N_1$, $N_2$, $N_3$, . . .) and “arrows” that are normally labeled by transformations (*GMIT*, definition 9.2.1, p. 195). The “legality” of a certain configuration of nodes and arrows is determined *solely* by the group or semigroup structure of its transformations; at this point, we need no information about which musical objects the nodes contain. A “transformation network,” however, requires an assignment of musical objects to nodes in manner consistent with the transformations (definition 9.3.1, p. 196). Although not predetermined by mathematical rules, the particular nodes and arrows chosen and the assignment of musical objects to nodes are meant to reflect meaningfully on actual musical events in the piece. The visual display of information in a transformation network is also a key component of transformational analysis that is not captured by its mathematical definition (see fig. 8.12 [*GMIT*, p. 191], which has an implicit chronological and pitch height dimension). Lewin considers these questions in *GMIT* (pp. 212–17) and in the second chapter of *MFT*, as discussed below. He dis-

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14. The serial operations RICH, TCH, etc. (*GMIT*, pp. 180–88) generate groups that do not act transitively on the set of series (i.e., not every pair of series is connected by some transformation), while the group of Klang-operations (pp. 175–80) acts transitively but not simply transitively.

tunguishes between “formal” networks, such as the pitch-class circle, that faithfully represent the complete structure of a group or semigroup action (that is, potential transformations), and “figural” networks, such as those in MFT and chapters 7–10 of GMIT, that depict only actual transformations in the case at hand (MFT, pp. 47–53). Although it is clear that Lewin’s transformational analysis favors figural networks, definitions 9.2.1 and 9.3.1 are neutral on this issue.16

One of Lewin’s most revealing transformational analyses constitutes the second chapter of MFT, “Making and Using a Pcset Network for Stockhausen’s Klavierstück III” (MFT, pp. 16–67). Tonal theory has well-established “musical spaces” illustrating harmonic relationships (the circle of fifths, the Tonnetz); in this essay, Lewin guides us through his attempts to uncover the “musical space” of an atonal piece. He takes great care to explain each step, including some wrong turns. In the process, we learn a great deal more about transformational theory—and Lewin himself—than about Stockhausen. This essay represents neither Lewin’s best writing (it is untidy and sometimes digressive) nor his most significant musical analysis (Klavierstück III is sixteen measures long). Yet I enjoy it immensely: it is so rich with ideas, so full of the sense of Lewin as a teacher, performer, and listener—as a person. Anyone whose experience of music involves a sense that “there is a story to be told” in the world of abstract—even mathematical—relationships will feel an enormous sympathy for Lewin’s response to this piece.

In listening to and playing Klavierstück III, Lewin developed “a sense that there is an overall story to be told in the establishment of a certain consistent harmonic field for the piece, and in the progression of the piece through that field” (p. 67). Lewin’s harmonic field revolves around the transpositions and inversions of the pentachord \{A♭, A, B♭, B, D\}. His goal is to draw a network—a “compactly organized little spatial configuration” (p. 17)—symbolizing both the abstract relationships between the pentachords used and the actual progressions realized in Klavierstück III. He begins by listing, in chronological order, the instances of the pentachordal set class that he hears in the piece, rejecting some forms identified by Harvey17 because he does not hear them. In example 2.7 (p. 42), he abstracts the succession of pentachord forms as an ear-training exercise for the reader. He proceeds to uncover the abstract

16. I would argue that the existence of a group action becomes less essential in figural networks—in fact, many of Lewin’s musical analyses are perfectly understandable without group structure. In the Stockhausen analysis (MFT, chap. 2), it is more important to know that J-inversion commutes with transposition than to know that both operations belong to a group acting on every form of the set class \{0, 1, 2, 3, 6\}. In addition, I submit that the set of durations, acted on by augmentation and diminution (GMIT, ex. 2.2.5, pp. 24–25), could label nodes and arrows in a meaningful figural network, although constructing a formal network would require a way of “diminishing” a zero-time interval.

relationships between the pentachords listed. Several consecutive inversionally related pentachords fall into pairs sharing their chromatic tetrachord; he refers to such pairs as “J-associates.” The mathematical rules for combining J-inversion and transposition follow (pp. 25–30). (The fact that there is a GIS structure lurking in the background is not the primary thrust of the analysis, however.) Lewin’s first network reflects the chronology of the piece and some of the transformational relationships between the pentachord forms (p. 32). His second depicts a space through which the music moves (p. 34). Although both networks are well formed mathematically, Lewin is happier with the layout of the second, which he judges “tell[s] a better story, both because the sequence of events moves within a clearly defined world of possible relationships, and because—in so moving—it makes the abstract space of such a world accessible to our sensibilities” (p. 41). He stops short of further levels of abstraction: he notes that eleven “missing” pentachord forms could be added, revealing theoretical possibilities that do not actually appear in the piece. However, he does not consider them appropriate for his analysis of Klavierstück III.

Lewin’s network analysis of the Tarnhelm and Valhalla themes of Das Rheingold (GMIT, pp. 175–80) also tells an interesting story—not only because of the musical insight it offers but also because he later revised his analysis in a 1992 article and in GMIT’s new preface (pp. xiii–xv). He employs three operations belonging to the group of “Riemannian” operations acting on the set of major and minor triads, or “Klangs.” All three are musically distinct; however, the group to which they belong does not act on Klangs simply transitively—this structure is not “formally ‘intervallic’ ” (p. 180). He comments in his 1992 article, “To sum the matter up, there is no unique Riemann-type relationship abstractly specified by the notion of starting at G-flat major and arriving at B-flat minor; the system makes a number of transformations conceptually available, each of which abstractly carries G-flat major to B-flat minor. In mathematical language, one says that the pertinent group of transformations ‘is not simply transitive.’ Wagner interweaves such multiple relationships with particular craft.” As in the Stockhausen analysis, the visual organization of a transformational network is significant. Lewin redraws the respective networks in the preface to make the similarities between the Tarnhelm and Valhalla themes more obvious.

18. This is a “tree falling in the forest” paradox: all possible p- and P-forms do not appear in the piece, so we cannot really “know” what transformations are appropriate in every situation. Perhaps this is why Lewin resists adding nodes to his network.


20. Ibid., 54.
Reflection on *GMIT* and *MFT*

Let us return to the questions posed at the beginning of this review.

*Is the mathematics correct?* In situations where Lewin defines his terms precisely, his mathematics is sound, with a few exceptions. John Clough’s 1989 review of the first edition of *GMIT* identified minor mathematical errors in pages 53, 55, 58, and 59 (these have not been corrected in the new edition). Moreover, figures 9.14 (a) and (b) (*GMIT*, p. 213) and 9.16 (p. 216) do not satisfy the requirements of a “well-formed” graph in the sense of definition 9.3.1: the result of applying the “relative” transformation followed by the “parallel” transformation to a Klang is equivalent to transposing the Klang by $T_9$ *only if* the Klang is major. Lewin’s correction of a similar situation in figure 8.2 (p. 179) in the new preface suggests that he may have intended all the graphs in chapter 9 to be “well formed.” If this is the case, figures 9.14 and 9.16 are in error. If not, then figures 9.14 and 9.16 show that Lewin used networks in more general contexts than definition 9.3.1 implies; however, he should have made this point clear.

*Do generalized interval systems and transformations model “our” intuitions about abstract musical structures in a sufficient number of situations?* In general, applied mathematics is “messy” in that one cannot expect theoretical


22. Although these lapses seem minor, they actually point to a general problem with transformation networks: the requirement that the relationships between transformations expressed by a graph be *universally* true precludes quite a bit of profitable musical analysis. Julian Hook (“Cross-type Transformations and the Path Consistency Condition,” *Music Theory Spectrum* 29 [2007]: 1–39) proposes that we consider networks that are consistent in the case at hand as well as those that are universally consistent; his analysis of two passages in Rimsky-Korsakov’s *Christmas Eve* demonstrates the advantages of his proposal. Although Hook does not point it out, his less restrictive condition does not strictly require transformations to act on every object in the musical space—a relaxation endorsed by Dmitri Tymoczko (“Generalizing Musical Intervals”). John Rahn (“The Swerve and the Flow: Music’s Relationship to Mathematics,” *Perspectives of New Music* 42 [2004]: 130–49) proposes to allow multiple arrows between nodes in a transformation graph. The benefits of this are more aesthetic than mathematical, as it is already possible for two arrows to connect the same pair of objects in a transformation network, because the same two objects can appear multiple times in a network.

23. It is not always clear to whom Lewin’s “we” refers. He repeats the phrase “we intuit” ten times on pages 25 and 26 of *GMIT* (for example, he asserts on page 25, “If $i$ and $j$ are intervals . . . we intuit being able to compose them”). Mathematicians often use “we” to mean the author and the reader. A mathematician might say “Let us assume that parallel lines never intersect,” thus alerting the reader that she is to adopt this axiom for the time being. To say, “We intuit that parallel lines never intersect” attributes this subjective judgment to the reader (it turns out that allowing parallel lines to intersect, as longitude lines on the globe do, has been quite fruitful mathematically). Moreover, Lewin sometimes states that “we hear” certain musical relationships in a piece (see p. 145 of *MFT*, for example). I feel somewhat bullied in this situation. (Of course,
structures to transfer well from one situation to another.\textsuperscript{24} It is unrealistic to expect that such an ambitious program will apply across musical styles or even in different musical works—at the very least, this question should be addressed head-on and with a good deal of caution. Although Lewin’s vision of an abstract musical space (or spaces) is brilliant, he sometimes seems too reluctant to examine his models critically. In general, I find transformational models more satisfactory than the strict GIS construction. In the following few paragraphs, I summarize some key passages in which I found myself hungry for more discussion.

In the second chapter of \textit{GMIT}, Lewin refers back to his picture of an arrow (see Fig. 1) and states that he intends to generalize “our intuitions about the \textit{i}-arrow” beyond intervals between pitches or pitch classes (p. 16). On pages 25 and 26, he gives what seems a persuasive argument that intervals “intuitively” form a group. This is a strong statement. Although groups are general structures, they are also extremely rigid ones, and groups are \textit{not} the structures most commonly studied by applied mathematicians. It is one thing to describe a precise situation and prove that it represents a group. It is another thing to assert that the group model is sufficient to represent our intuitions in most situations. Although all but one of the examples he has introduced in \textit{GMIT} so far are groups, “our intuitions” in particular cases are irrelevant: a structure either is a group or is not a group (one would hardly say that “I intuit that the real numbers form a group”). Therefore, I assume that “our intuitions” as represented on pages 25 and 26 are to be generalized to other musical contexts. With this in mind, I take issue with this passage on two counts:

First, let us examine one of the statements on page 26, the truth of which is necessary for intervals to form a group: “We intuit the composition \textit{ij} of the intervals \textit{i} and \textit{j} to be itself an interval of the system.” Figure 2 represents this situation. It seems reasonable to assume that combining the interval from \textit{s} to \textit{t} (\textit{i} in the figure) with the interval from \textit{t} to \textit{r} (\textit{k}) produces the interval from \textit{s} to \textit{r} (\textit{ik}), although there are counterexamples. However, I do not intuit that there is always a natural way to compose intervals between two different pairs of objects (that is, to compose the interval from \textit{s} to \textit{t} with the interval from \textit{p} to \textit{q}). The obstruction is that, in some reasonable situations, there exist intervals that may originate with some objects but not with others. Lewin’s example 2.2.5 (the set of durations, measured in nonnegative numbers with some specified unit beat [\textit{GMIT}, pp. 24–25]) illustrates this problem: A duration of one beat is two beats less than a duration of three beats ($\text{int}(3, 1) = -2$), and a duration of ten beats is six beats less than a duration of sixteen beats ($\text{int}(16,$

\textsuperscript{24} Quite a few computer scientists study music in a more “applied” fashion. Any paper on music information retrieval demonstrates a striking contrast to the tidiness of Lewin’s constructions.

\textsuperscript{24} Lewin may have been using the “editorial we,” but as he also uses “I” frequently, I assume this is not the case.)
What, then, is the combination of \( \text{int}(3, 1) \) and \( \text{int}(16, 10) \)? If we follow the recipe of Figure 2, we must apply the interval “six beats less” to the duration of one beat—that is, we must find a duration that is six beats less than one beat. As Lewin comments, “We can not conceive . . . a duration lasting precisely five units less than no time at all” (\( GMIT \), p. 30).\(^{25}\) (Lewin’s extension of transformational theory to semigroups later in \( GMIT \) somewhat remedies this situation, but still does not address examples like 2.2.5 where reasonable transformations do not apply to every object.\(^{26}\)

Second, I do not intuit that there must be exactly one interval between any two objects, as the GIS structure requires (“\( \text{int} \) is a function” [p. 26]). As is clear in later chapters of \( GMIT \), this “intuition” applies only in the narrow set of circumstances that produce a GIS. The last few chapters present quite a few situations in which there are multiple operations or transformations between pairs of objects (see fig. 9.14, p. 213) or none at all. Several of these transformations (the Riemannian operations and the serial permutations RICH, TCH, etc.) actually do belong to groups, although groups whose actions are not simply transitive (the group of serial transformations is not even transitive). Moreover, Lewin later emphasizes that multiple structures—GISes or otherwise—exist on the same set of objects: “we intuit several or many musical spaces at once” (\( GMIT \), p. 250).

\(^{25}\) The same problem arises when we consider undirected intervals: we can diminish a perfect fifth, but not a unison (and, moreover, as intervals between pitch classes, a perfect fifth is equivalent to a perfect fourth—perhaps there are \( \text{two ways to diminish a perfect fifth} \)). Tymoczko offers a cogent criticism of Lewin on this point (“Generalizing Musical Intervals”). “Geometrical music theory” offers one solution: to model the space whose objects are undirected intervals between pitch classes as a line segment extending from unison to the tritone (see Clifton Callender, Ian Quinn, and Dmitri Tymoczko, “Generalized Voice-Leading Spaces,” \( \text{Science} \) 320 [2008]: 346–48; and Dmitri Tymoczko, “The Geometry of Musical Chords,” \( \text{Science} \) 313 [2006]: 72–74). Like the line segment, many geometrical spaces have “boundaries” that prohibit a GIS or transformational structure. The “diminish” transformation encounters the boundary of its respective geometrical space.

Although geometrical music theory does challenge Lewin’s ideas, what strikes me is how Lewinnian its constructs actually are. A few geometrical musical spaces are GISes; others employ a “tangent space” construction that can be modeled by a topological group acting on the space. In general, however, the action of that group is neither simply transitive nor defined at every point of the space—boundaries and so-called singular points create problems.

\(^{26}\) Lewin offers to “salvage” example 2.2.5 with the GIS of example 2.2.6, in which he defines a “measure” and deems that two beats belong to the same beat class if they are separated by a whole number of measures. Is this solution reasonable? For one thing, the objects of examples 2.2.5 (durations) and 2.2.6 (beat classes) are fundamentally different in a way that pitches and pitch classes are not (the intervals of 2.2.6—“directed durations”—are more like the objects of 2.2.5). Moreover, measuring durations by directed differences in beat class has a natural interpretation in music with repeated rhythmic patterns but limited use in other contexts. In addition, not all music—not even all the music Lewin analyzes—has a consistent time signature or, indeed, any time signature. I am more comfortable with example 2.2.5 as it stands.
Are Lewin’s musical analyses persuasive? The final four chapters of GMIT and all of MFT are more firmly grounded in the analysis of actual musical works than the first part of GMIT. Here, Lewin allows the mathematical structure to follow from his musical analysis. This is Lewin’s strength: he vividly illustrates how mathematical thinking aids study of these pieces. I am particularly impressed by how rarely he strays far from music into the world of pure mathematics. Although he develops quite a bit of abstraction, he always motivates his constructions with examples. This is what all good applied mathematics does: it develops the mathematical tools appropriate for the case at hand. Although nonintervallic networks are “messy” in that the identification of a transformation is sometimes subjective, they are the appropriate tools to use in many analyses.

Lewin’s kind of analysis has quite a different flavor from more holistic treatments of musical works. Consider how Lewin contrasts his own analysis of Stockhausen’s Klavierstück III with that of Nicholas Cook (MFT, pp. 62–63).27 Much that Cook discusses is missing from Lewin’s analysis: Lewin makes little mention of registration, contour, rhythm, and dynamics—all features that are more immediately audible in the piece than the progression of pentachords. (For that matter, Lewin does not place Stockhausen in a wider historical context or mention his other works.) Although he admits that Cook’s analysis is more comprehensive and practical, Lewin takes great issue with Cook’s seeming belief that Klavierstück III can be understood without any special effort on the part of the listener and his implication that the piece has no “consistent harmonic field” (MFT, p. 67). In a particularly revealing

Lewin’s point that we should strive to be the listeners his analysis demands is well taken. That said, I do not find all of Lewin’s analyses persuasive. His insistence on eschewing the pitch transposition model for pitch-class transposition does not distinguish between transpositions that are immediately audible because they occur in the pitch domain (for example, many, but not all, of the repeated transpositions in the last movement of Brahms’s Horn Trio, Op. 40 ([GMIT, p. 161]) and those that require a highly trained ear and repeated listening (for example, the transpositions and J-inversions of Klavierstück III). 29

Consider Debussy’s “Feux d’artifice,” measures 87–88 (MFT, p. 107). The double glissandi descend thirty-eight semitones from pitch classes A and A♭ to G and G♭. In order to preserve a GIS structure, Lewin analyzes the glissandi as T₁₂. Surely, an audible descent of more than three octaves creates a dramatic contrast to the T₁ in the following measure. Furthermore, to be mathematically consistent, Lewin should have labeled the glissandi T₁₀. Using the pitch labels {A₅, A♭₅} and {G₂, G♭₂} is one solution; however, the GIS of pitch transpositions does not recognize the close relationship between T₃₈ and T₂. A more elegant solution is to posit a “non-intervallic” T₃₈ gesture from {A, A♭} to {G, G♭}, as in Transposition group 2 above. Transposition group 2 would also be useful in an analysis of Brahms’s Horn Trio (GMIT, pp. 165–69). It both distinguishes the repeated T₃₄ transformations G↓→F₅→E₃ (mm. 33–35) and B₄→A₄→G₄ (mm. 137–141) from the T₅₄ followed by T₂ gesture of E♭₃→D♭₆→C♭₆ (mm. 123–125) and reveals the


29. I confess that I cannot hear these; although I am happy to take Lewin’s word that he can, the scientist in me would like to know more: Was Lewin able to hear the pentachords without looking at the score? Before looking at the score? If not, did he consider that problematic?
similarities between all three transformations, which Lewin analyzes as repeated instances of $T_{10}$. Transposition group 2 also avoids some of the awkward discussions of registration in the analysis of Webern’s Opus 10, no. 4 (MFT, pp. 69–71). Lewin has developed perfectly good tools (nontransitive group actions) for such situations, and yet he does not use them here.

Setting aside any considerations of music, how does GMIT hold up as a math book? Although the book is for the most part technically correct, I am not particularly impressed with GMIT as mathematical writing. (This criticism is in no way meant to detract from the depth, power, and sophistication of Lewin’s ideas.) Lewin is not always clear about the boundaries between science and art. I take no issue with subjective judgments—meaningful musical analysis should have a subjective component—but the danger is that a novice may conclude that Lewin’s analysis is always determined by “scientific” rules. In quite a few instances, his strict mathematical definitions are not sufficient to represent his intuitions about music. This issue surfaced in my discussions of distance in a GIS and of the aesthetic conventions of transformational networks. Moreover, the conflict between definition 9.3.1 and figures 8.2 (of the first edition), 9.14, and 9.16 in GMIT suggests that Lewin’s intuition and his definition of a transformational network may have been at odds. I do not fault Lewin’s musical intuitions, but rather his mathematical ones—the mathematical definition of a transformational network seems too restrictive. In contrast, if Lewin meant to use GISes to model “distance” or “measurement,” their definition is incomplete. He writes, “We do not really have one intuition of something called ‘musical space.’ Instead, we intuit several or many musical spaces at once. GIS structures and transformational systems can help us to explore each one of these intuitions, and to investigate the ways in which they interact, both logically and inside specific musical compositions” (GMIT, p. 250). I would emphasize the word “help” in the last sentence.30

Although there is no question that reading Lewin is worth the effort, I find some features of the books annoying. Most of the proofs in GMIT are routine and should have been left as exercises for the reader, with solutions in an appendix (e.g., pp. 46–59). Lewin indicates that the reader can skip the proofs (and I encourage this), but the problem is that valuable intuitive explanations—“what’s really going on here”—are often either buried in clutter or omitted. Moreover, the rule in good mathematical writing is “notation, notation, notation.” Some terms in GMIT are used precisely and consistently, others—in particular, “interval,” “transposition,” and “transformation”—are not. Definition 3.4.1 (p. 46) deems the “transpose” of a musical element by an

30. Hook’s review of GMIT and MFT (“David Lewin and the Complexity of the Beautiful”) contains a summary of related developments in the field since the publication of GMIT. I would add to his list the burgeoning field of computational music analysis, which has some connection to the statistical methods of Lewin’s Generalized Set Theory (GMIT, chaps. 5–6).
interval \( i \) ("interval" meaning a group element) to be the unique element of
the musical space that lies at interval \( i \) from \( s \), thus leaving no word to uniquely
signify transposition in pitch or pitch class.\(^{31}\) However, after pages of discus-
sion in which "interval" and "transposition" are used in this extended sense,
his comments in chapter 8 imply that the reader has forgotten that "intervals"
and "transpositions" are group elements and actions (p. 178). As I noted in
note 12 above, "transformation" seems to refer to a member of a simply tran-
sitive group in a key passage—the more precise term would be "operation."\(^{32}\)
In addition, Lewin redefines several standard mathematical terms, substituting
"family" for "set," while using "set" to mean "finite subset of a family." He
also defines "operation," "transformation," and "into" in unorthodox ways
and uses words such as "measurement" imprecisely. He sometimes changes
his own notation—for example, he defines "the J-inversion" differently in
each chapter of \( MFT \). The complete lack of bibliographic references to mathe-
matical literature sets a bad precedent (happily, more recent mathematical
music theory has not resisted connecting to mathematics). In addition, al-
though one cannot fault Lewin for this, the mathematical typesetting of
\( GMIT \) is atrocious, resulting in cluttered pages that further obscure his mean-
ing. Moreover, the lack of section titles and chapter subheadings on every
page makes navigation of \( GMIT \) troublesome, as the chapter titles are generic
(e.g., Transformation Graphs and Networks (1), (2), (3), and (4)).

While I applaud Oxford’s decision to reprint Lewin's books, some aspects
of the new edition of \( GMIT \) are disappointing. Complete chapter headings
and a few notes about errata would have been welcome additions; a glossary
and a new layout by an experienced mathematical typesetter would have been
a blessing. Moreover, Hook points out that a few errors have been introduced
on the pages that have been modified in the new edition.\(^{33}\) In addition, page
numbers differ from the first edition in the introduction but have not been
updated in the index.

Lewin was a revolutionary thinker, and \( GMIT \) is a revolutionary book. It
is a book that—to paraphrase Lewin's comments on Stockhausen (\( MFT \),
p. 62)—challenges, provokes, and in some ways infuriates me. As with many
books that are revolutionary in their field—Newton’s \( Principia \), Darwin’s
\( Origin of Species \)—it has its flaws. The very newness of its ideas is the source of

\(^{31}\) He sometimes uses "formal transposition" to refer to an action that is not a "transposi-
tion" in the historical sense. However, he does not make this distinction consistently. Moreover,
definition 3.4.1 concerns the GIS case, not pitch transpositions acting on pitch-class space.

\(^{32}\) For those who care about groups, it is worth noting that every set of operations belongs
to some group, while transformations that are not operations cannot. A collection of transforma-
tions acting on a set can fail to belong to the interval group of a GIS for two reasons: (1) it con-
tains a transformation that is not an operation, and thus does not belong to a group, or (2) all its
members are operations, but at least two of the operations act identically on the elements of the
set. It is worth keeping this straight.

some of these flaws: in response to the challenge of constructing a music theory relevant to tonal as well as atonal music, Lewin created much of his notation from scratch, building on foundational concepts such as “interval,” “transposition,” and “transformation.” Lewin’s reputation as one of the most brilliant and influential music theorists of his generation is firmly established and justly deserved. The relevant question for theorists today is not, however, “What did Lewin mean by such-and-such a statement?” but “What is the best way to model such-and-such a musical situation?” Lewin’s methods are often—but not always—sufficient to model our intuitions about abstract musical spaces; more importantly, his ideas are always worth engaging.

RACHEL WELLS HALL


As the title indicates, music is just one of three areas under study in this book, along with the social history of religion and the history of Pietism. Tanya Kevorkian, as a social historian, is conscious of the ways her approach to musicology differs from that of a more traditional musicologist: rather than studying scores, music treatises, correspondence, diaries, or appointment and payment records, Kevorkian looks at sources on pewholding, on appointment of clerics, and on the roles of councilors. These, she says “can illuminate musical life in ways that traditional musicological sources cannot” (p. 10). One should not read this book expecting to encounter new source materials about Baroque musicians. Johann Sebastian Bach may in some sense be the central character in the book, but Kevorkian’s contribution to Bach studies lies in adding to our knowledge of the context in which Bach lived and worked, not in offering original interpretations of Bach’s life or works.

Her introduction addresses the use of the term baroque, which, except in the field of music, is usually associated more with Catholicism than Protestantism. Yet, as Kevorkian shows, the movement crossed denominational boundaries. Protestant Leipzig, with its world-class trade fair and thriving merchant class, developed a flourishing cosmopolitan culture of art, architecture, luxury goods, coffeehouses, pleasure gardens, and opera in the late seventeenth and early eighteenth centuries. While the church in many ways adapted to the new values of polite society, the Pietist movement emerged among those who regarded this trend as a movement away from true Christian values.

Most original of the topics addressed by Kevorkian is the chapter on pewholding. Based on detailed study of archival records, the author allows the reader unusual insights into the physical hierarchy maintained in Leipzig