Entrepreneurial Risk Aversion, Net Worth Effects and Real Fluctuations

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Abstract

This paper examines the combined effect of asymmetric information and private entrepreneurial risk aversion on investment decisions. The standard optimal debt contract becomes modified by the introduction of insurance and a risk premium that entrepreneurs demand due to the uncertainty of their investment returns: the private equity premium. In general equilibrium, the private equity premium may become a mechanism that magnifies the effects of shocks. A structural estimation of the model’s parameters using Chilean and U.S. data shows that the entrepreneurial risk aversion assumption has more empirical relevance in an economy where smaller privately-held businesses are relatively more prevalent than where the corporate sector predominates, like the U.S.

Keywords: Entrepreneurial risk aversion; Asymmetric and private information; Contracts; Business cycles
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1. Introduction

The fact that emerging markets tend to present stronger output responses to shocks than industrialized economies has been widely studied in the literature. Backus & Kehoe (1992) and Prasad, Agenor & McDermott (1999), to name a few, provide empirical evidence supporting such an assertion. More often than not, differences in observed average GDP volatility among countries are attributed to variations in the levels of development of their financial markets. In some economies, however, the entrepreneurial sector can be a significant source of volatility as well.

Table C.1, which presents the volatility of entrepreneurial activity across countries as measured by the standard deviation of the Total Entrepreneurial Activity index (TEA),\(^2\) shows that the volatility of entrepreneurial activity is on average more than 2.5 times larger in developing countries than in developed ones. An illustrative example is the Chilean case. While it boasts one of the most robust financial systems in its region, its economy still reacted strongly to the effects of the Asian crisis of the late 1990s. A reason often cited for the observed response was that the entrepreneurial sector in Chile moved quickly from an early-to-mid 1990’s boom euphoria to a deep depression in the following years.

While numerous models have attempted to explain business cycle fluctuations as a result of financial imperfections, few incorporate the behavior of the entrepreneurial sector as a direct mechanism behind business cycles volatility. Figure C.1 provides preliminary evidence of a positive simple direct correlation between the total entrepreneurial activity volatility and output growth volatility, as measured by the standard deviations of the TEA index and per capita real GDP growth for 31 countries, respectively.

This paper deals with the role of the private entrepreneurial sector as an additional source of business cycle volatility. As in Bernanke & Gertler

\(^2\)This index, developed by the Global Entrepreneurship Monitor (GEM), measures the relative amount of nascent entrepreneurs and business owners of young firms for 75 countries from 2001 to the present (van Stel, Carree & Thurik, 2005).
(1989), capital-producing entrepreneurs finance their investments by engaging in debt contracts with lenders, a relationship that is subject to standard frictions such as asymmetric information problems. In addition, this paper introduces a second friction in this model that results from private entrepreneurial risk aversion.

For simplicity, entrepreneurs are commonly modeled as risk-neutral agents. As Gale & Hellwig (1985) point out, “risk neutrality is not an unreasonable assumption to make in the case of investors since it can be justified as a consequence of risk-pooling. However, the authors also claim that risk neutrality “makes less sense in the case of entrepreneurs and indeed is merely a ’simplifying’ assumption which should be relaxed if possible.” That is, as entrepreneurs are less likely to have access to complete risk-pooling for their idiosyncratic risks, risk aversion is arguably a more realistic assumption in modeling their investment decisions. In fact, Moskowitz & Vissing-Jørgensen (2002) show that private entrepreneurs tend to invest in privately-owned businesses that are typically small and owned by few or just one entrepreneur. The authors also find that private entrepreneurs usually invest at least 50 percent of their assets in a single company, and thus are highly vulnerable to project-specific, uninsurable risk.

The analysis in this paper uses a set-up similar to Carlstrom & Fuerst (1997) and Bernanke, Gertler & Gilchrist (1999) in that entrepreneurs invest in capital projects using both their own net worth and borrowed funds, and maintain the projects true ex-post capital return as private information. Agency problems arise from the positive probability that projects can fail, which are also costly for the lender as the revenues after default cannot be fully recovered. As a result, lenders optimally charge an external finance premium.

Introducing risk aversion for entrepreneurs modifies the contract in two ways. First, risk-averse entrepreneurs demand insurance in order to guarantee a minimum level of consumption. Therefore, production of capital also incorporates the insurance cost. Second, the overall risk premium incorporates not only the aforementioned external finance premium and insurance costs, but also an extra premium that risk-averse entrepreneurs internalize due to the stochastic nature of their investment returns, to which is referred in this paper as the private equity premium. The equilibrium price that final goods firms pay to rent capital from entrepreneurs depends on the combined interaction of these premiums.
In a general equilibrium framework, the private equity premium may become a mechanism that further magnifies the effects of real shocks. Specifically, any shock that, say, reduces entrepreneurial profits and wealth not only impacts the standard external finance premium, but it also reduces demand for insurance by entrepreneurs. Consequently, entrepreneurs’ effective risk aversion and thus the private equity premium increase. As a response, all else equal, entrepreneurs optimally reduce their supply of capital, which in equilibrium results in both an increase in the rental rate of capital and a drop in the demand for productive capital from final goods firms. As a result, output, entrepreneurial profits and net worth fall in subsequent periods, generating similar additional rounds of adjustment. Consequently, the resulting dynamics of this model suggest that economies that present relatively larger shares of privately-held businesses, all else equal, should show more output volatility than economies with a relatively more important corporate sector.

This work differs from other models that deal with risk aversion and financial imperfections. For instance, unlike Angeletos & Calvet (2006), this paper includes entrepreneurs that endogenously determine the amount of self-insurance. This model is also different from Rampini (2004) in that financial imperfections, such as default risk, play a crucial role in affecting business cycles fluctuations. Like in Meh & Quadrini (2004), optimal contracts cannot provide full insurance. However, entrepreneurs self-insure through contingent instruments in their paper, while entrepreneurs purchase insurance directly from lenders in this paper. This difference is particularly relevant as the dynamics of insurance is the main driving force behind the private equity premium and capital investment.

The outline of this paper is as follows. Section 2.1 examines the optimal entrepreneur-lender interaction in partial equilibrium. Section 2.2 embeds the optimal contract into a stochastic dynamic general equilibrium model and analyzes the dynamics resulting from real shocks. Section 3 presents calibration and simulation exercises. Section 4 presents an empirical analysis of the model’s main implications. Section 5 provides concluding remarks.

2. The Model

2.1. The Contract

This section briefly describes the main attributes of the modified optimal contract. There are two groups of participants in this contracting relationship. First, risk-averse entrepreneurs, who invest in capital production ($K_{t+1}$)
that firms rent at a gross rental rate $R_{t+1}$ in order to produce final goods. The second group is made up of risk-neutral competitive lenders, who provide financing to entrepreneurs and are paid a contracted interest rate ($Z_{t+1}^j$) for this service.

As in Bernanke & Gertler (1989), entrepreneurs finance their investments using both internal net worth ($N_t^j$) and borrowing ($B_{t+1}^j$).

\[ B_{t+1}^j = K_{t+1}^j - N_t^j \]  

That is, to purchase raw capital, the entrepreneur borrows from lenders an amount $B_{t+1}^j$.\(^3\)

The main source of uncertainty arises from the unknown amount of effective capital that results from an entrepreneur choosing $K_{t+1}^j$ units of raw capital. Namely, the actual quantity of capital available for production is given by $\omega^j K_{t+1}^j$, where $\omega$ is an stochastic variable with $E_t(\omega) = 1$ and known distribution. There is asymmetric information as the realization of $\omega$ is initially observed only by the entrepreneur. That is, the contract between entrepreneurs and lenders is subject to informational frictions in that the ex-post stochastic return of the investment ($\omega^j$) is private information and idiosyncratic to each entrepreneur. As in Townsend (1979), costly state verification implies that lenders can only learn the true return to capital by incurring an auditing cost.

An optimal contract between a risk-averse entrepreneur and a risk-neutral lender under the informational friction framework described above must be one that maximizes the entrepreneur’s expected utility subject to the lender participation constraint. In particular, for non-default states (defined as the values of $\omega^j$ that are higher than a certain cutoff, $\hat{\omega}_j$), the contract must be equivalent to a standard risk-neutral contract. Namely, due to costly state verification, there is no monitoring, the entrepreneur maintains the realization of $\omega^j$ as private information and optimally repays the lender the contractual gross interest rate, $Z_{t+1}^j$. That is, when $\omega^j > \hat{\omega}_j$, the entrepreneur receives the upper part of the distribution of stochastic capital profits, $\omega^j R_{t+1} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j$.

For low states of nature ($\omega^j < \hat{\omega}_j$), however, the standard debt contract is suboptimal as any solution must ensure the risk-averse party a positive

\(^3\)As proved in Appendix A, when risk-free assets are protected by limited liability, a debt contract with maximum equity participation weakly dominates any other optimal contract.
consumption level in any state. This result naturally occurs since in default states monitoring takes place and information becomes observable by both parties, and therefore, it would be optimal for the risk-neutral agent to face the uncertainty. Consequently, lenders optimally provide the entrepreneur with a fixed payment by insuring, say, a fraction \( x^j_t \leq 1 \) of the entrepreneur’s net worth.\(^4\) Therefore, the optimization problem can be characterized by entrepreneurs maximizing the following expected utility:\(^5\)

\[
\int_{0}^{\hat{\omega}^j} U(x^j_t N^j_t) \, dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U(\omega^j R_{t+1} K^j_{t+1} - Z^j_{t+1} B^j_{t+1}) \, dH(\omega) \quad (2)
\]

which contains both a certain component (the insurance \( x^j_t N^j_t \)) and a stochastic component that entrepreneurs receive only when the realization of \( \omega^j \) is higher than the default cutoff value, \( \hat{\omega}^j \).

Since an entrepreneur would be indifferent between defaulting or not when the utility levels of both strategies are equal, then the default cutoff value, \( \hat{\omega}^j \), solves \( x^j_t N^j_t = \hat{\omega}^j R_{t+1} K^j_{t+1} - Z^j_{t+1} B^j_{t+1} \). As a result, the entrepreneur’s expected utility can be re-expressed as

\[
\int_{0}^{\hat{\omega}^j} U(x^j_t N^j_t) \, dH(\omega) + \int_{\hat{\omega}^j}^{\infty} U(x^j_t N^j_t + (\omega^j - \hat{\omega}^j) R_{t+1} K^j_{t+1}) \, dH(\omega) \quad (3)
\]

That is, the entrepreneur receives a minimum guaranteed consumption level \( x^j_t N^j_t \) regardless of the realization of \( \omega^j \). In addition, if \( \omega^j > \hat{\omega}^j \), the entrepreneur obtains an extra random revenue \( \omega^j R_{t+1} K^j_{t+1} \) and repays a fixed amount \( \hat{\omega}^j R_{t+1} K^j_{t+1} \).

The insurance is an instrument through which the entrepreneur can partially hedge against the projects idiosyncratic risk. Note that incomplete

\(^4\)Note that the only source of uncertainty that this model considers is idiosyncratic. In addition, as in Carlstrom & Fuerst (1997), I assume for simplicity that entrepreneurs can maintain enough anonymity that it is possible to engage in one-period contracts, regardless of their repayment history. Note that this is a simplifying assumption that allows us to focus the models attention on the aggregate effects of entrepreneurial risk aversion, even though it does not deal with other related issues like dynamic strategies and reputation. Therefore, throughout this paper, the existence of an optimal contract is conditional on these assumptions.

\(^5\)In the general equilibrium model, as in Bernanke et al. (1999), entrepreneurial consumption is assumed to be a constant fraction of the entrepreneurs final wealth. This constant rate has been omitted here as it does not affect the maximization problem.
insurance is an endogenous feature of this model: entrepreneurs transfer the risk to risk-neutral agents in low states and optimally face uncertainty in high states by maintaining the realization of \( \omega \) as private information. That is, if \( \omega < \hat{\omega} \) (default), the risk is borne by the lender, while when \( \omega > \hat{\omega} \) (repayment), the risk is assumed by the entrepreneur. Therefore, the default cutoff \( \hat{\omega} \) captures the extent to which risk is ex-ante shared between lenders and entrepreneurs.

In addition, note that default is not equivalent to business failure or bankruptcy in the sense of inability to repay. Instead, default represents those states of nature in which the entrepreneur is not willing to face uncertainty and thus prefers to obtain insurance, even if the entrepreneur were able to repay the debt. Put differently, default corresponds to those states such that the entrepreneur believes that the expected present discounted value of future returns do not compensate for the price that the entrepreneur would receive if he or she liquidated the company's assets. In such a case, the entrepreneur would transfer the risk to other parties by obtaining insurance. In reality, such insurance may take multiple forms, such as the price at which a private equity fund would purchase the entrepreneur's business or the value at which the entrepreneur would sell the company to shareholders if an IPO were to be conducted. Without loss of generality, the buyer in this model is the lender.\(^6\)

The lender participation constraint is then given by:

\[
\int_{0}^{\hat{\omega}} [(1 - \mu) \omega R_{t+1} K_{t+1}^j] dH(\omega) + \int_{\hat{\omega}}^{\infty} [\hat{\omega} R_{t+1} K_{t+1}^j] dH(\omega) - x_i^j N_t^j \geq (1 + \rho_{t+1}) B_{t+1}^i \tag{4}
\]

which requires that expected revenues from lending net of default costs (as captured by the parameter \( \mu \) above) must be no lower than the opportunity cost of funds, where \( \rho_{t+1} \) is the economy’s risk-free interest rate. In equilibrium, due to default costs, lenders require compensation above the opportunity cost of their funds: the external finance premium.

The optimal contract minimizes agency costs and is given by the joint

\(^6\)Note that this assumption is not relevant. Even if a third party were the actual buyer, in the absence of additional frictions, the dynamic analysis of the model would remain unchanged. Namely, the purchaser of the entrepreneurs project would assume the risk of the business, pay the auditing costs and be in charge of repaying the debt, for which an external finance premium would be charged by the lender.
choice of \( K_{t+1}^j, \hat{\omega}^j \) and \( x_t^j \) that maximizes the entrepreneur’s expected utility subject to the lender participation constraint, taking as given for now variables that are predetermined as of period \( t \), such as net worth and the rental rate of capital.\(^7\)

This problem’s optimality conditions are captured by:

\[
\int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) R_{t+1} dH(\omega) = \left[ 1 + \frac{E(U'(\cdot)|\omega < \hat{\omega})}{E(U'(\cdot)|\omega > \hat{\omega})} \right] \times \\
\left[ (1 + \rho_{t+1}) - (1 - \mu) \int_0^{\hat{\omega}} \omega R_{t+1} dH(\omega) - R_{t+1} \hat{\omega} \left[ 1 - H(\hat{\omega}) \right] \right]
\]

\[\left(5\right)\]

\[
\int_{\hat{\omega}}^{\infty} \omega R_{t+1} dH(\omega) - \left(1 + \rho_{t+1}\right) B_{t+1}^j
\]

\[\left(7\right)\]

which after some simple algebraic manipulation, imply the following condition:

\[
\frac{x_t^j N_t}{K_{t+1}} + \frac{[1 - H(\hat{\omega})]}{K_{t+1}} \int_{\hat{\omega}}^{\infty} \omega R_{t+1} dH(\omega) = \left[ 1 - H(\hat{\omega}) \right] \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1} dH(\omega)
\]

\[\left(8\right)\]

Equation (8) states the problem’s optimal condition: in equilibrium, investment decisions equate the entrepreneur’s expected marginal revenues and marginal costs. Expected marginal revenues are given by the sum of the insurance per unit of capital and the expected net return to capital that the

\(^7\)Gale & Hellwig (1985) analyze an equivalent contracting problem between a risk-averse entrepreneur and risk-neutral lenders and prove that such a contract is optimal. The authors employ a general implicit monitoring cost function that is a positive function of both the state and the investment level. In this model, given that the monitoring cost function, \( \mu \omega R_{t+1} K_{t+1}^j \), is a positive linear function of both the state \( \omega \) and the level of investment \( (K_{t+1}^j) \), all properties from Gale and Hellwig’s framework also apply here, and thus the contract proposed in this section is also optimal.
entrepreneur obtains in non-default states, as captured by the first and second terms of the left-hand side of equation (8), respectively.

The marginal cost of investing is, in turn, captured mostly by three components. The first two are standard: the unit cost of capital repayment in non-default states and the opportunity cost of investing the entrepreneur’s net worth, as given by the first and second terms of the right-hand side of equation (8), respectively. A risk-averse entrepreneur, however, faces a third cost associated with the fall in utility from facing uncertain returns. This cost is captured by the covariance between the stochastic capital returns ($\omega$) and the entrepreneur’s marginal utility of consumption, which is negative for any strictly concave utility function and is given by the third term on the right-hand side of equation (8). This additional cost borne by the risk-averse entrepreneur is called in this paper the private equity premium.\footnote{The argument of $U''()$ is the same as the argument of the entrepreneur’s utility function (equation (3)). In addition, there is an extra term in equation (8), $\varphi_t = \mu \omega h(\hat{\omega}) \int_{\hat{\omega}}^{\infty} (\omega - \hat{\omega}) R_{t+1} dH(\omega)$, which captures the change in expected default costs due to changes in $\hat{\omega}$ and whose value is negligible.}

To sum up, for a given demand for capital from final goods firms, the equilibrium rental rate of capital and the stock of capital are affected by both the external finance premium and the private equity premium. Therefore, the existence of risk-averse entrepreneurs, all else equal, translates into higher expected return to capital and lower capital investment by entrepreneurs than under the assumption of risk neutrality. Further, as analyzed in Section 2.2, the endogenous private equity premium in general equilibrium creates an additional mechanism that causes business cycle fluctuations to respond more strongly to real shocks.

2.2. Aggregate Effects

This section examines the aggregate impact of entrepreneurial risk aversion by incorporating the modified supply of capital introduced in the previous section, into a standard stochastic dynamic general equilibrium framework.

Consider an economy that produces and consumes a single good in an infinite horizon framework. The supply of capital from entrepreneurs is determined by the first order conditions of the optimal contract (equations (5), (6) and (7)). Aggregate demand for capital, on the other hand, is determined by a representative firm that produces the good by combining labor
(L) supplied by workers, and capital (K) supplied by entrepreneurs, through a standard constant returns to scale Cobb-Douglas production function

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]  

where \( A_t \) is an aggregate multifactor productivity factor.

The firm maximizes profits by optimally choosing capital, labor and total output. The firm’s profits are zero in equilibrium. The resulting standard optimality conditions for capital and labor are given by, respectively:

\[ r_t K_t = \alpha Y_t \]  
\[ W_t L_t = (1 - \alpha) Y_t \]

where \( r_t \) is the net rental rate of capital and \( W_t \) is the labor wage rate.

Capital demand and supply jointly determine the economy’s optimal capital investment and rental rate of capital. The supply labor comes from the representative worker that maximizes lifetime utility over consumption and leisure,

\[ E_{t-1} \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{1}{\upsilon} L_t^\upsilon \right] \]

subject to the budget constraint

\[ C_t + D_{t+1} = W_t L_t + (1 + \rho_t) D_t \]

where \( \upsilon > 0 \) is the elasticity of labor supply, \( C \) is household consumption and \( D \) are deposits held at financial intermediaries earning risk-free interest rate \( \rho_t \).

The representative worker’s optimality conditions yield standard Euler equation and labor supply condition:

\[ \frac{1}{C_t} = \beta(1 + \rho_{t+1}) E_t \left[ \frac{1}{C_{t+1}} \right] \]  
\[ \frac{W_t}{C_t} = L_t^{\upsilon - 1} \]

In aggregate terms, the total amount of raw capital is both part of entrepreneurs’ net worth \( N \) and financed externally \( B \). Therefore, analogous to equation (1):
The evolution of the aggregate capital stock $K_{t+1}$ follows:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

where $I$ is the aggregate investment demand and $\delta$ denotes the capital depreciation rate.

From equations (3) and (4), the accumulated aggregate equity of the entrepreneurial sector from renting capital to operating firms ($V_{t+1}$) is given by:

$$V_{t+1} = R_{t+1}K_{t+1} - (1 + \rho_{t+1})B_{t+1} - \mu \int_{0}^{\omega} \omega R_{t+1}K_{t+1} dH(\omega)$$

where the term $\mu \int_{0}^{\omega} \omega R_{t+1} dH(\omega)$ captures both the default and insurance costs.

In order to simplify the dynamics of aggregate net worth, I assume a framework of overlapping generations of entrepreneurs that live for two periods. As schematized in Figure C.2, an entrepreneurial generation is born in period $t - 1$ and receives a bequest $N_{1}^{t-1}$ (the subscript denotes the period and the superscript indicates the age of entrepreneurs). This generation invests its endowment (plus an amount $B_t$ of borrowing) in a risky project that yields a level of aggregate equity of $V_{t}^2$ in the following period.

I assume that entrepreneurs subsequently consume fraction $1 - \eta$ of their equity and bequeath the rest to future generations of entrepreneurs. Bequests are put into a common pool so that all entrepreneurs from the same generation receive the same bequest. That is, there is *ex-ante* homogeneity among entrepreneurs in that every new generation receives a bequest $N_{t}^{1} = \eta V_{t}^{2}$ and makes the same investment decisions. Heterogeneity among entrepreneurs emerges from the *ex-post* idiosyncratic realization of capital returns ($\omega$). This environment is a consequence of the one-period contacting analysis introduced in section 2.1, which in turn implies intertemporally separable entrepreneurial generations that bequeath an exogenous fraction $\eta$ of their wealth to anonymous future generations. Such a parameter is
empirically determined in this paper by both calibration and structural estimation. In addition, this framework avoids having entrepreneurial initial wealth, on top of idiosyncratic risk, as an additional source of heterogeneity in this model. At the same time, this specification allows us to account for the constant creation and destruction of firms, and would avoid the possible scenario that entrepreneurs may accumulate a sufficiently high level of wealth that borrowing is not needed for investments. Such a situation would obviate the need for this discussion.

Consequently, entrepreneurial net worth and consumption in period $t$ can be defined respectively as

$$N_t = \eta \left\{ (1 + \rho_t) N_{t-1} + \left[ R_t - (1 + \rho_t) - \mu \int_0^{\hat{\omega}} \omega R_t dH(\omega) \right] K_t \right\} \tag{19}$$

$$C_t^E = \frac{1 - \eta}{\eta} N_t \tag{20}$$

The model closes by imposing market clearing conditions that must be satisfied in each period. For the goods market, production equals the sum of the purchase of new capital and consumption goods in each period:

$$Y_t = I_t + C_t + C_t^E \tag{21}$$

For the financial market, workers’ savings must equal the total amount of borrowing demanded by entrepreneurs:

$$D_t = B_t \tag{22}$$

Finally, the technology shock parameter ($A$) follows an autoregressive process

$$A_t = \phi A_{t-1} + \varepsilon^A_t \tag{23}$$

where $\varepsilon^A_t$ is an independent and identically distributed innovation with $\varepsilon^A_t \sim N(\mu_A, \sigma^2_A)$.

Therefore, the risk-averse rational expectation stochastic dynamic general equilibrium is defined by equations (5), (6), (7), (9), (10), (11), (13), (14), (15), (16), (17), (19), (20), (21), (22) and (23) that solve for $Y_t$, $L_t$, $K_{t+1}$, $I_t$, $R_t$, $\hat{\omega_t}$, $W_{t+1}$, $x_t$, $N_t$, $N_{t+1}$, $B_{t+1}$, $D_{t+1}$, $C_t$, $C_{t+1}$, $C^E_t$ and $A_t$.

[FIGURE C.3 HERE]
The effect of a shock on this economy and its impact and interaction with the external finance premium and private equity premium are schematically summarized in Figure C.3. A shock that negatively affects entrepreneurs’ profits \( (V_t) \) reduces the amount of net worth available for investment. Subsequently, a lower \( N_t \) is translated into lower levels of investment, since agency problems lead to a higher external finance premium, as discussed in section 2.1. The reduced level of capital translates into lower entrepreneurial profits, further reducing net worth in subsequent periods. In addition, a drop in net worth also impacts entrepreneurial risk-aversion frictions described in section 2.1, as lower \( N_t \) decreases the level of insurance \( x_t N_t \). A decrease in entrepreneurs’ minimum guaranteed consumption increases their effective level of risk aversion and thus raises the private equity premium. As a consequence, capital decreases again for a given \( R_{t+1} \), further amplifying the initial effect of the shock.

3. Numerical Analysis

3.1. Calibration

This section presents the values of the parameters used to numerically simulate this model. I select some parameter values in a standard fashion. For instance, following Bernanke et al. (1999), the share of capital in production \( (\alpha) \) is set at 0.35, the discount factor \( \beta \) is 0.99, the quarterly depreciation rate \( (\delta) \) is 0.025 and the labor supply elasticity is set at 3 (which implies a value for the parameter \( \nu \) equal to \( 4/3 \)). In addition, the idiosyncratic return \( \omega \) is assumed to be log-normally distributed with a mean of one and standard deviation of \( \sigma_\omega \). As common in the literature, I initially set the constant relative risk-aversion coefficient \( (\gamma) \) equal to 2.

I choose the remaining parameters so that they imply steady-state values that match previously estimated measures for the Chilean economy. In particular, the values for the volatility of capital return parameter \( (\sigma_\omega) \), the default cost rate \( (\mu) \) and the entrepreneurial saving rate \( (\eta) \) that match the empirically observed estimates of the default rate \( H(\hat{\omega}) \), the overall risk premium \( R - (1 + \rho) \) and the ratio of capital to net worth \( K/N \) at the steady state. Using data from the Central Bank of Chile, I estimate that the average lending interest rate in the period 2002-2008 is about 212 basis point over the risk-free rate. The \( K/N \) ratio I obtain from Fernandez (2005), who finds a debt-to-equity ratio of firms of 0.612, which is equivalent to \( K/N = 1.612 \).
Finally, Calvo, Drexler, Flores & Pacheco (2009) obtains a default rate on business loans of about 9% for 2001.

The corresponding values of the parameters $\sigma_\omega$, $\mu$ and $\eta$ for the risk-averse case are 0.07, 0.02 and 0.948, respectively. One may observe that, as expected, in order to match the benchmark values for the risk premium, default rate and $K/N$ ratio under risk aversion, it was necessary to establish a much more favorable setting than under risk neutrality, especially in terms of default costs and volatility of returns. This setting implies an excessively large steady-state value for the insurance rate $x_t$, and therefore, particularly low exposure to uncertainty and private equity premium, as observed in Table C.3. For these reasons, I also analyze two other alternative scenarios: scenario I is the one described above; scenario II allows for a higher $K/N$ ratio; and scenario III, where the risk-premium can be larger as a consequence of the private equity premium. That said, the corresponding values of the parameters $\sigma_\omega$, $\mu$ and $\eta$ are, respectively, 0.033, 0.025 and 0.815 for scenario II and 0.1325, 0.095 and 0.85 for scenario III. Table C.2 summarizes the values for $\sigma_\omega$, $\mu$ and $\delta$ that I obtained from the calibration procedure.

Table C.3 presents the steady state values resulting from the calibration of the model. Details on the derivation of the unique steady state are included in Appendix B. Some results worth noting include the fact that under the alternative scenarios for the risk-averse case, the insurance rate $x_t$ is in fact lower than in scenario I. Also, the steady state level of entrepreneurial net worth ($N$) is lower than in the risk-neutral case. This result makes sense as risk aversion more closely fits the behavior observed for privately-held businesses, which are typically smaller.

3.2. Simulation

This section explores the effects of a small aggregate shock on the main variables of the model. In particular, a one-time, one-percent, unexpected

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9For the risk-neutral benchmark case, I obtain $\sigma_\omega = 0.31$, $\mu = 0.132$, and $\eta = 0.953$. All these values are within the range of previous estimates.

10Pagano, Panetta, & Zingales (1998) show that independent private firms tend to decrease their financial leverage substantially after they become public.
increase in aggregate multifactor productivity is considered. The technology shock is standard and persists with an autocorrelation coefficient equal to 0.95.

[FIGURE C.4 HERE]

The analysis is carried out by examining the impulse-response functions that result from solving the complete system around its unique deterministic steady state.\textsuperscript{11} The continuous lines denote the risk-aversion case that is incorporated through an endogenous private equity premium, while the dashed lines represent the benchmark risk-neutral case. Figures C.4, C.5 and C.6 show the resulting impulse response functions for output, capital, consumption, the private equity premium and other variables for scenarios I, II and III, respectively.

Following the shock, capital and output rise as a result of greater productivity, where the increase in output is stronger for the risk-averse case. This amplification effect occurs as expected: in the short term, the rise in entrepreneurial profits and net worth as a response to the shock produces an increase in the entrepreneur’s guaranteed consumption level \((x_t N_t)\), which in turn decreases the private equity premium. This increase takes place in addition to the standard effects of changes in wealth on the agency costs, as captured by the external finance premium. Consequently, capital investment increases more than in the risk-neutral benchmark case, and thus the stock of capital and output also rise above and beyond the case where there is no private equity premium. The sharp increase in investment reduces the initial consumption level in the economy and leads workers to raise their labor supply. As a consequence, income and, therefore, consumption increase in later periods.

[FIGURE C.5 HERE]

[FIGURE C.6 HERE]

\textsuperscript{11} Since this model incorporates the entrepreneur problem’s first order conditions and lender participation constraint from the optimal contract in section 2.1 into the general equilibrium model, a log-linear approximation is sufficient to capture all second-order effects of risk aversion, in particular the private equity premium.
Note that the amplification feature described above is not as strong for scenario I as for scenarios II and III. As explained earlier, this result arises from the fact that the steady state equilibrium consistent with matching the aforementioned values for the default rate, risk premium and the $K/N$ ratio is found for exceptionally low levels of the bankruptcy cost rate and volatility. As noted in section 2.1, the existence of a risk premium relies on the existence and magnitude of asymmetric information in general and on uncertainty in particular for the risk-averse case.

The effect of the shock dies out over time as higher capital supply leads to lower rental rates of capital and entrepreneurs’ profits, as observed in equation (19). Naturally, this behavior affects the private equity premium in the opposite direction and translates into a subsequent faster drop in capital and output back to the steady state over time. That is, though the private equity premium implies that shocks may be further amplified over the business cycle, those effects tend to be less persistent than in the risk-neutral case.

Table C.4 shows the standard deviation and coefficient of autocorrelation as measures of the volatility and persistence of output, consumption and investment. As one may observe, higher levels of volatility are generated in the economy with risk-averse entrepreneurs, as the private equity premium amplifies real shocks over time. In addition, faster convergence to the steady state values is translated into lower persistence of the effects of shocks for risk-averse entrepreneurs relative to risk-neutral entrepreneurs.

One nice feature of this model is that it is able to generate countercyclical risk premia. Carlstrom & Fuerst (1997) saw that a drawback of their model was the resulting procyclical behavior of the risk premium. Their response occurs because the demand-induced increase in capital (due to the rise in productivity) more than offsets the supply-induced increase in capital as a consequence of the drop in agency costs. They further point out that “this effect could be overcome if entrepreneurial net worth would rise more sharply in the period of the shock.” In this model, not only is net worth rising more sharply, but also the rise in net worth increases the supply of capital due to both the standard alleviation of agency costs and to the drop in the private equity premium. In the net, this response allows for a mild drop in the rental rate of capital $R_{t+1}$ past the first period and thus in the overall risk premium (in particular, in scenario III).
4. Empirical Evidence

The objective of this section is two-fold: (i) to estimate structural parameters of the model introduced in this paper; (ii) to compare the results obtained for the Chilean economy with those for the U.S. economy, as the role of risk-averse entrepreneurs in creating sharper business cycle fluctuations is likely to be more important in economies where privately-owned businesses represent a relatively larger fraction of the real sector (as in Chile) when compared to economies where the corporate sector is more important (like the U.S.). Therefore, the null hypotheses are that the estimated risk-aversion coefficient ($\gamma$) is significantly (i) greater than zero for the Chilean economy, and (ii) higher than that of the U.S.

I conduct the estimation of the model’s main parameters using the Bayesian method. This estimation procedure uses observed data and the Bayes theorem in order to update “prior” beliefs for the parameters to be estimated and their probability distributions. The results are Bayesian estimators of the posterior distribution of the model’s parameters that result from minimizing a posterior expected loss function.

Besides the entrepreneurial risk-aversion coefficient ($\gamma$), the parameters I estimate are the variance of capital returns ($\sigma^2_\omega$), the default cost rate ($\mu$), the entrepreneurial saving rate ($\eta$), along with the shock parameter ($\sigma_A$) and its autoregressive coefficient ($\phi_A$).

4.1. Priors

The prior distributions for the estimated parameters reflect what is known about them in terms of their probability distributions, means and standard deviations before observing the empirical data. Most of the prior mean values were obtained in the calibration procedure presented in section 3.1 and Appendix C. For the risk-aversion coefficient ($\gamma$), for instance, I use the normal distribution around the initially assumed mean value of 2 for both countries, with a fairly high standard deviation in order to favor higher flexibility in the estimation.

The prior values for the mean and standard deviation of the remaining parameters were chosen specifically to each country and according to calibration. The parameters $\mu$ and $\sigma^2_\omega$ were assumed to follow an inverse gamma distribution.

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12I perform an equivalent calibration procedure for the U.S. economy as well, which is presented in Appendix C
distribution in order to ensure positive estimates. The standard deviations for these variables were set at values equal to roughly one half of their prior means. Finally, the entrepreneurial saving rate ($\eta$) follows a normal distribution with rather low standard deviations in order to avoid the possibility of saving rates greater than one. Finally, I use the uniform distribution for the standard deviation ($\sigma_A$) and persistence ($\phi_A$) of the shock parameter, with lower and upper bounds equal to 0 and 1, respectively.

4.2. The Data

I use quarterly time series data on output to conduct the estimations, which I obtain from the Central Bank of Chile and the U.S. Bureau of Economic Analysis (BEA), for Chile and the U.S., respectively. The data series range from 1986.1 to 2010.1 for Chile and from 1947.1 to 2010.1 for the U.S. In addition, I detrend the series using the Hodrick-Prescott filter in order to transform the series in terms of their percentage deviations from their trends.

4.3. Results

Table C.5 shows the results for the Bayesian estimations. For the coefficients other than that of entrepreneurial risk-aversion, the values are mostly not significantly different between the two countries or with respect to their prior values. The exceptions include the variance of capital returns ($\sigma^2_\omega$) and the shock parameters ($\sigma_A$ and $\phi_A$). As expected, there is higher volatility in capital returns in Chile than in the U.S. In addition, the shock is stronger in Chile than in the U.S., however less persistent. The difference in volatility and persistence of real shocks towards stronger but less persistent values for less developed economies is consistent with previous literature.\textsuperscript{13}

Also as expected, the risk-aversion coefficient is statistically significantly higher for Chile than for the U.S. economy. In particular, $\gamma$ is positive and statistically significantly different from zero for Chile, while not for the U.S. That is, this model requires a higher risk aversion coefficient ($\gamma$) in order to explain the responses to shocks followed by the Chilean economy than by the U.S. economy.

\textsuperscript{13}For instance, Mendoza (1995) finds that terms of trade shocks are on average almost three times more volatile however less persistent in emerging markets relative to developed countries.
To sum up, empirical results support the idea that risk-neutrality may in fact be a reasonable assumption for the U.S. economy, but not for economies like Chile. That is, risk aversion on the part of entrepreneurs may not only make sense in theory for some economies, but also shows empirical relevance, given that Chile presents a higher share of privately-held businesses relative to corporations than the U.S. Therefore, this result seems to indicate that entrepreneurial risk aversion may be realistic and statistically significant in explaining output fluctuations for economies that are more heavily composed of privately-held businesses but not for economies where the corporate sector is relatively more important.

5. Concluding Remarks

Evidence on the wide variation of business cycle fluctuations among countries facing similar shocks is often almost entirely attributed to financial imperfections. However, evidence also suggests that economies enjoying both healthy financial systems and robust private sectors may still respond strongly to shocks and move rapidly from booms to recessions, and vice versa. This paper suggests that such economic behavior could be explained in part by risk aversion on the part of private entrepreneurs, as this sector may be an additional source of amplification of shocks.

By examining the microeconomic impact of entrepreneurial risk aversion in the context of an optimal contract under asymmetric information, I show that (i) the total capital rental cost incorporates the private equity premium or the premium required by risk-averse entrepreneurs due to the uncertainty associated with their investment returns; (ii) entrepreneurial risk aversion limits the economy’s capital supply through the private equity premium; and (iii) the private equity premium is countercyclical.

In the general equilibrium analysis, we observe that the private equity premium may further amplify shocks over the business cycles. Entrepreneurs respond by increasing their supply of capital to final goods firms, producing a positive impact on capital, output and consumption. That is, a time varying demand for insurance by entrepreneurs helps explain the countercyclical behavior of the risk premium and thus provide an additional reason for procyclical entrepreneurial activity. Consequently, considering risk-averse entrepreneurs helps explain in part the magnitude of business cycle fluctuations that in some economies information frictions alone have failed to rationalize.
As shocks are further amplified when private entrepreneurs are assumed to be risk averse, this model predicts that economies with a relatively important private entrepreneurial sector should show more volatile business cycles than economies with a private sector composed largely of publicly traded companies. That is, two economies with similar financial health and real sector robustness may present significantly different business volatility due to differences in the ownership structure of the productive sector. The structural estimation of the risk-aversion coefficient for Chile and the U.S. tends to support this prediction.

Policy implications of this model include that improvements in information technology and transparency in the privately-owned private sector may help alleviate some of the extra volatility produced by asymmetric information. Policies encouraging more established businesses to become public or the further expansion of private equity firms would accomplish similar effects.

Acknowledgments

I thank John Shea, Luis Felipe Céspedes, Enrique Mendoza, Nicolás Magud, Michael Pries and seminar participants at the University of Maryland, the Interuniversity Student Conference (IUSC) at the University of Pennsylvania, University of Chile, Catholic University of Chile and Trinity University for insightful comments, suggestions and encouraging conversations out of which this product emerged. All remaining errors are mine.

References


Appendix A. Proof of the Maximum Equity Participation Property

The lender participation constrain can be re-expressed as

\[
\lambda_t^j x_t^j N_t^j = \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1}^j K_{t+1}^j dH(\omega) + (1 - \mu) \int_{0}^{\hat{\omega}} \omega R_{t+1}^j K_{t+1}^j dH(\omega) - (1 + \rho_t) [K_{t+1}^j - \lambda_t^j N_t^j]
\]  

(A.1)

Replacing equation (A.1) into the objective function, we get the following unconstrained optimization problem

\[
\int_{0}^{\hat{\omega}} U \left\{ \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1}^j K_{t+1}^j dH(\omega) + (1 - \mu) \int_{0}^{\hat{\omega}} \omega R_{t+1}^j K_{t+1}^j dH(\omega) - (1 + \rho_t) [K_{t+1}^j - N_t^j] \right\} dH(\omega)
\]

\[
+ \int_{0}^{\hat{\omega}} U \left\{ (\omega^j - \hat{\omega}^j) R_{t+1}^j K_{t+1}^j + \int_{\hat{\omega}}^{\infty} \hat{\omega} R_{t+1}^j K_{t+1}^j dH(\omega) + (1 - \mu) \int_{0}^{\hat{\omega}} \omega R_{t+1}^j K_{t+1}^j dH(\omega) - (1 + \rho_t) [K_{t+1}^j - N_t^j] \right\} dH(\omega)
\]

which is independent of \( \lambda_t^j \).□

Appendix B. The Steady State

The steady state representation of equations (9), (10), (11), (13), (15), (17), (16), (19), (20), (21), and (22), respectively, is given by:

\[
Y = AK^{\alpha}L^{1-\alpha} \quad (B.1)
\]

\[
rK = \alpha Y \quad (B.2)
\]

\[
WL = (1 - \alpha)Y \quad (B.3)
\]

\[
C = WL + \rho D \quad (B.4)
\]

\[
WL = CL^{\nu} \quad (B.5)
\]

\[
I = \delta K \quad (B.6)
\]

\[
N + B = K \quad (B.7)
\]
\[ N = \eta \{ (1 + \rho)N + RK - (1 + \rho)K - \mu RK \int_0^{\hat{\omega}} \omega dH(\omega) \} \]  
(B.8)

\[ C^E = \frac{1 - \eta}{\eta} N \]  
(B.9)

\[ Y = I + C + C^E \]  
(B.10)

\[ D = B \]  
(B.11)

plus the steady state optimal capital supply of this economy, as derived from the optimal contract (equations [5], [6] and [7]).

Let us define the private equity premium \((PEP)\) at the steady as the difference between the steady state rental rate of capital \((R)\) and the sum of the risk-free interest rate \((1 + \rho)\) and the external finance premium, as given by the lender default cost \(\mu R \int_0^{\hat{\omega}} \omega dH(\omega)\). Therefore, equation (B.8) can be re-expressed as:

\[ \left[ \frac{1 - \eta(1 + \rho)}{\eta} \right] = \frac{K}{N} \cdot PEP \left( \frac{K}{N} \right) \]  
(B.12)

As shown in section 2.1, the private equity premium depends primarily on the capital to net worth ratio \(\kappa = K/N\). That is, proportional changes to both capital and net worth, all else equal, do not affect the private equity premium. That said, given the values for \(\eta\) and \(1 + \rho\), a unique steady-state \(\kappa\) ratio and thus \(PEP\) and \(R\) can be identified from the equation above.

From equation (B.2), we can obtain the following relationship between \(K\) and \(Y\):

\[ K = \left( \frac{\alpha}{r} \right) Y \]  
(B.13)

Using this last equation along with equations (B.3), (B.4), (B.7) and (B.11), one can express consumption in the steady state as a function of output and net worth:

\[ C = \left[ 1 - \alpha + \frac{\rho \alpha}{r} \left( 1 - 1/\kappa \right) \right] Y \]  
(B.14)

The next expression for labor in the steady state results from solving for \(L\) from equations (B.3) and (B.5):

\[ L = \left[ \frac{(1 - \alpha)Y}{C} \right]^{1/\nu} \]  
(B.15)
Finally, from equations (B.14) and (B.15), one gets the following expression that uniquely determines the value of \( L \) in the steady state:

\[
L = \left( \frac{1 - \alpha}{1 - \alpha + \frac{\rho \alpha}{\tau} (1 - 1/\kappa)} \right)^{1/\nu}
\]  

(B.16)

The unique steady state values of the remaining variables of the model can be easily obtained.

Appendix C. Calibration for the U.S. Economy

A calibration procedure for the U.S. economy equivalent to the one for Chile is also performed. As in Bernanke et al. (1999), the average risk premium between the prime lending rate and the six-month Treasury bill rate is about 200 basis points, and the \( K/N \) ratio is 2. Previous measures of the default rate vary from 0.974 percent (Fisher, 1998) to 3 percent (Bernanke et al., 1999). I use 2 percent as the middle value.

[TABLE C.6 HERE]

Table C.6 summarizes the values for the parameters \( \mu, \eta \) and \( \sigma_\omega \) that are necessary to match the steady state values for \( \kappa, H(\hat{\omega}) \) and \( R - (1 + \rho) \) presented above for both the risk-neutral case and scenario I of the risk-averse case.
Figure C.1: Scatter plot and linear fit of the volatility of GDP growth on volatility of entrepreneurial activity
Figure C.2: Overlapping Generation of Entrepreneurs
Figure C.3: External Finance Premium and Private Equity Premium
Figure C.4: Effects of a Positive Productivity Shock (scenario I)
Figure C.5: Effects of a Positive Productivity Shock (scenario II)
Figure C.6: Effects of a Positive Productivity Shock (scenario III)
<table>
<thead>
<tr>
<th>Country</th>
<th>Std. dev.</th>
<th>Country</th>
<th>Std. dev.</th>
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<tr>
<td>Belgium</td>
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<td>Finland</td>
<td>0.0151</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.0063</td>
<td>United States</td>
<td>0.0155</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.0082</td>
<td>South Africa</td>
<td>0.0161</td>
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<tr>
<td>Norway</td>
<td>0.0090</td>
<td>Greece</td>
<td>0.0162</td>
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<td>Switzerland</td>
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<td>Ireland</td>
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<td>Germany</td>
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<td>Brazil</td>
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<td>Slovenia</td>
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<td>Netherlands</td>
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<td>China</td>
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<td>Spain</td>
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<td>Denmark</td>
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<td>India</td>
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<td>Japan</td>
<td>0.0134</td>
<td>Mexico</td>
<td>0.0501</td>
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<tr>
<td>France</td>
<td>0.0151</td>
<td>Peru</td>
<td>0.0821</td>
</tr>
</tbody>
</table>

Advanced economies* 0.0121
Emerging economies 0.0316

Note: *As defined by the IMF (International Monetary Fund, 2012)

Source: Global Entrepreneurship Monitor (GEM)

Table C.1: Entrepreneurial volatility across countries
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>share of capital to output</td>
<td>0.35</td>
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<tr>
<td>$\gamma$</td>
<td>coefficient of risk aversion</td>
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<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.99</td>
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<tr>
<td>$\nu$</td>
<td>labor supply elasticity parameter</td>
<td>$4/3$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
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Risk-Neutral Case

<table>
<thead>
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<th>Value</th>
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<td>$\mu$</td>
<td>default cost rate</td>
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<td>$\eta$</td>
<td>entrepreneur saving rate</td>
<td>0.9589</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>volatility of returns</td>
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</table>

Risk-Averse Case - Scenario I

<table>
<thead>
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<tr>
<td>$\eta$</td>
<td>entrepreneur saving rate</td>
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<tr>
<td>$\sigma_\omega$</td>
<td>volatility of returns</td>
<td>0.0852</td>
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Risk-Averse Case - Scenario II

<table>
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<th>Value</th>
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</thead>
<tbody>
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<td>$\mu$</td>
<td>default cost rate</td>
<td>0.0208</td>
</tr>
<tr>
<td>$\eta$</td>
<td>entrepreneur saving rate</td>
<td>0.9254</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>volatility of returns</td>
<td>0.0571</td>
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</table>

Risk-Averse Case - Scenario III

<table>
<thead>
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<th>Value</th>
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</thead>
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<td>$\eta$</td>
<td>entrepreneur saving rate</td>
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<td>$\sigma_\omega$</td>
<td>volatility of returns</td>
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Table C.2: Calibration Chile
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<td>$\kappa$</td>
<td>capital to net worth ratio</td>
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<tr>
<td>$H(\hat{\omega})$</td>
<td>default rate</td>
<td>9%</td>
</tr>
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<td>$R - (1 + \rho)$</td>
<td>risk premium</td>
<td>2.1%</td>
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<tr>
<td>$\left(\frac{WL}{C}\right)$</td>
<td>share of labor income to consumption</td>
<td>93.84%</td>
</tr>
<tr>
<td>$\left(\frac{I}{Y}\right)$</td>
<td>investment to output ratio</td>
<td>27.9%</td>
</tr>
<tr>
<td>$\left(\frac{C}{Y}\right)$</td>
<td>consumption to output ratio</td>
<td>69.27%</td>
</tr>
</tbody>
</table>

Risk averse case - Scenario I

| $\kappa$  | capital to net worth ratio         | 1.6   |
| $H(\hat{\omega})$ | default rate          | 9%    |
| $R - (1 + \rho)$ | risk premium          | 2.1%  |
| $\left(\frac{WL}{C}\right)$ | share of labor income to consumption | 93.82% |
| $\left(\frac{I}{Y}\right)$ | investment to output ratio       | 27.9% |
| $\left(\frac{C}{Y}\right)$ | consumption to output ratio      | 69.28%|
| $x$        | insurance rate              | 85.35%|

Risk averse case - Scenario II

| $\kappa$  | capital to net worth ratio         | 3.61  |
| $H(\hat{\omega})$ | default rate          | 9%    |
| $R - (1 + \rho)$ | risk premium          | 2.1%  |
| $\left(\frac{WL}{C}\right)$ | share of labor income to consumption | 88.86% |
| $\left(\frac{I}{Y}\right)$ | investment to output ratio       | 22.15%|
| $\left(\frac{C}{Y}\right)$ | consumption to output ratio      | 58.07%|
| $x$        | insurance rate              | 77.62%|

Risk averse case - Scenario III

| $\kappa$  | capital to net worth ratio         | 1.6   |
| $H(\hat{\omega})$ | default rate          | 9%    |
| $R - (1 + \rho)$ | risk premium          | 7.79% |
| $\left(\frac{WL}{C}\right)$ | share of labor income to consumption | 97.7% |
| $\left(\frac{I}{Y}\right)$ | investment to output ratio       | 9.23% |
| $\left(\frac{C}{Y}\right)$ | consumption to output ratio      | 61.73%|
| $x$        | insurance rate              | 75.05%|

Table C.3: Implied steady state values
<table>
<thead>
<tr>
<th></th>
<th>risk neutral</th>
<th>scenario I</th>
<th>scenario II</th>
<th>scenario III</th>
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</thead>
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<tr>
<td>$\sigma_Y$</td>
<td>0.0445</td>
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<td>$\sigma_C$</td>
<td>0.1812</td>
<td>0.1968</td>
<td>0.2254</td>
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<tr>
<td>$\sigma_I$</td>
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<tr>
<td>$\rho_Y$</td>
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<td>$\rho_C$</td>
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<td>$\rho_I$</td>
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<td>-0.0554</td>
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</tbody>
</table>

Note: $\sigma_i$ and $\rho_i$ denote the standard deviation and first-order coefficients of autocorrelation of simulated variables.

Table C.4: Theoretical Volatility and Persistence
<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Mean</th>
<th>Std. dev</th>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>$\gamma$</td>
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<td>2.0</td>
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<td>0.9343</td>
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<td>$\mu$</td>
<td>inv gamma</td>
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<td>0.01</td>
<td>0.0167</td>
<td>0.0077</td>
<td>0.0241</td>
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<td>$\eta$</td>
<td>gamma</td>
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<td>0.9225</td>
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<td>$\sigma_\omega^2$</td>
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<td>$\phi_A$</td>
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<td>1.0</td>
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<td><strong>278.862</strong></td>
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<th>Parameter</th>
<th>Distrib.</th>
<th>Mean</th>
<th>Std. dev</th>
<th>Mean</th>
<th>Std. dev</th>
<th>Confidence interval</th>
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<td>U.S.</td>
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<td>$\gamma$</td>
<td>normal</td>
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<td>-1.8998</td>
<td>0.2200</td>
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<td>inv gamma</td>
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<td>0.01</td>
<td>0.0192</td>
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<td>gamma</td>
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<td>0.9147</td>
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<td>inv gamma</td>
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Note: The intervals correspond to the 5th and 95th percentiles of the posterior distributions. Prior mean and standard deviation for uniform distribution correspond to the lower and upper bounds.

Table C.5: Bayesian Estimation
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<td>α</td>
<td>share of capital to output</td>
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<tr>
<td>γ</td>
<td>coefficient of risk aversion</td>
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</tr>
<tr>
<td>β</td>
<td>discount factor</td>
<td>0.99</td>
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<tr>
<td>ν</td>
<td>labor supply elasticity parameter</td>
<td>4/3</td>
</tr>
<tr>
<td>δ</td>
<td>depreciation rate</td>
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<table>
<thead>
<tr>
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<th>Risk-Neutral Case</th>
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</thead>
<tbody>
<tr>
<td>μ</td>
<td>default cost rate</td>
<td>0.132</td>
</tr>
<tr>
<td>η</td>
<td>entrepreneur saving rate</td>
<td>0.953</td>
</tr>
<tr>
<td>σω</td>
<td>volatility of returns</td>
<td>0.31</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Risk-Averse Case - Scenario I</th>
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</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>default cost rate</td>
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</tr>
<tr>
<td>η</td>
<td>entrepreneur saving rate</td>
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<tr>
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<td>volatility of returns</td>
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Table C.6: Calibration U.S.