INVESTMENT DECISIONS AND MARKET STRUCTURE UNDER INCOMPLETE INFORMATION

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ABSTRACT

This paper sheds some light on the factors behind the timing of investment decisions under asymmetric information. Firms receive signals through which they build beliefs on the state of demand. The timing of investment and the resulting market structure depends on how quickly uncertainty is resolved and the significance of asymmetries in information. Inefficient outcomes could result as firms may “suspect” stronger competition or if uncertainty leads them to postpone projects, due to the irreversible nature of investment. Simulations tend to confirm the differences in time to invest and market structure discussed for the cases analyzed in the model.

INTRODUCTION

Informational frictions as a source of inefficiencies in aggregate investment have been an extensive area of economic research. Since investment involves fixed costs and is usually irreversible, increased uncertainty may naturally cause investors to become more cautious and consider options like postponing investment decisions until some uncertainties are resolved. Under the context of the current economic crisis, for instance, it could make sense for potential investors to “wait” until there are clearer signs regarding, say, the extent to which the crisis may affect relevant variables, such as price and demand. In general, an investor would not only compare the present discounted value of investing versus its most profitable current alternative, but also the value of investing today versus the value of investing sometime in the future. That is, the investor will try to evaluate his or her “value of waiting.” (McDonald and Siegel, 1986)

The value of waiting can be affected by how quickly and effectively new relevant information arrives, which may include policy adjustments (interest rates, taxes, government programs, etc), changes in market conditions (demand and prices), changes in the structure of the industry (competition), and other factors. Note that information on competitors involves, for instance, observing action and inaction by other investors, as information rarely “arrives uniformly and comprehensively to every potential investor” (Caballero, 1999). That is, given that some information remains undisclosed during periods of inaction, the value of waiting gains relevance as the arrival of new information from others becomes more likely.

Consequently, incomplete information and/or information asymmetries may lead to suboptimal solutions, including postponing or not taking advantage of profitable investment opportunities. If this is the case, investments could evolve very slowly or even collapse (Caballero, 1999), potentially dragging down the rest of the economy. The aftermath of a recession is a common example of an environment in which uncertainty contributes to the
slowdown of investment decisions. In particular, firms may tend to postpone investment and hiring decisions during recessionary periods until concrete positive news arrives, thus delaying the recovery of the economy. This outcome could be particularly common in developing countries, where economic activity and investment, in particular, may exhibit stronger responses to shocks and, therefore, more volatility and uncertainty than more developed economies. Naturally, uncertainty about future economic conditions and ineffective normative and institutional conditions can further add to uncertainty. In addition, recessions are in many cases followed by the re-architecture of part of the labor market, capital market and fiscal position (Belke, 2009). Not surprisingly, many firms may decide to wait for stronger signals to confirm that it is a good idea to resume investments.

The flipside scenario, where investment grows very quickly in a short period of time, is also possible. That is, incomplete information could also lead to a decline in the value of waiting if other firms invest earlier and successfully capture a significantly large market share, thus potentially dramatically reducing the expected profitability of a project. That is, incomplete information could also imply the creation of incentives for firms to embark upon projects earlier than optimal. An illustrative case is that of periods of rapid technological progress, where numerous potentially attractive investment opportunities may present themselves to investors, though uncertainties on their profitability, feasibility and the structure of the competition may still be high.

One example of such a situation is the so-called “Dot com bubble” of the 1990’s. In its embryonic period, the general belief of high profitability in the information technology sector encouraged a large number of entrepreneurs to start new businesses over a short span of time. Uncertainty, however, remained high, including doubts about the resulting market structure. A few investors who entered early found profitable niches which allowed them to establish themselves and to obtain high market shares, recognition, efficiency and profitability. Others that entered later, and could not consolidate critical market shares, failed. Those who entered the market later and were able to survive, on the other hand, did so by offering newer, innovative or vastly improved services (e.g. Google, Netflix, Facebook, etc.).

A natural question is which of these two conflicting forces, at any point in time, will eventually prevail. Factors like uncertainty, unobservable preferences, competition, and expected returns, are likely to come into play in addressing this matter. The objective of this paper is to examine some of the factors that can affect the timing of investment decisions and the resulting market structure. In this framework, incomplete information could lead to inefficient equilibriums, in which investment projects may be executed either “too early” or “too late,” relative to the benchmark of perfect information.

Several papers have dealt with irreversible investment under uncertainty. Caplin and Leahy (1993), for instance, develop a macroeconomic model in which investors’ actions are suboptimal because they “fail to take into account the social value of the information that their investment reveals.” As information on the profitability of their industries is asymmetrically distributed among the participating investors, projects that succeed send positive signals to potential investors, who may respond by entering the industry. Equivalently, industries and economies will slowly recover from adverse shocks as inaction reduces the flow of hidden information. In Caplin and Leahy (1994), investors also possess private information but, due to adjustment costs in investment, will only respond to changes in information when they are highly convincing. Prior to that, however, information accumulates and no major changes in investment take place (“business as usual”), up until a certain threshold of information is reached (“market crash”). At that point, significant information is transmitted to other investors, who may respond accordingly (“wisdom after the fact”), usually producing magnified responses. Decisions by investors are also suboptimal, as they do not have incentives to share information with other investors.
Chamley and Gale (1994) develop a model in which a recession could last longer than it optimally would in equilibrium, as investors delay investment until they receive convincing positive information. Given that the higher the number of investors that decide to act, the more information is transmitted about the profitability of investments, investors have the incentive to be “the last to decide,” resulting in delays. This outcome occurs because the action of others only transmits information, but it does not affect the payoffs from investing. Finally, Drazen and Sakellaris (1995) introduce a model where the timing of investment decisions is affected by the likelihood of receiving new information at any point in time. The authors emphasize the distinction between uncertainty about the returns to investments and on when “uncertainty itself may be resolved.”

This paper adds to the existing literature by introducing the fact that investors may also face incentives to enter the market before it is optimal due to uncertainty about the resulting market structure. That is, as firms may exploit new markets or niches, they may temporarily enjoy monopolistic profits if they are active in the market before others. In particular, by using a simple monopolistic competition model with uncertainty regarding the true state of demand, a firm may earn positive profits if it enters the market alone. Profits would quickly fall when other firms enter the market after observing the realization of true demand, once the “first firm” starts selling. In this paper, firms also have incentives to wait as they will not invest before they have received convincing information about the state of demand. As firms receive random signals about the state of demand every period, one firm’s action and inaction reveals information about the number of positive signals it has received. Therefore, waiting may have the added benefit of investing with complete information.

This paper is structured as follows. In the next section, I comment about the method I employ in this paper. Next, I include a description of the model and how investors form their beliefs. The following section introduces and presents the implications of the benchmark case of no asymmetries in information. The fourth part conducts the equivalent analysis but for the realistic case where there is asymmetry in information, including uncertainty about the other firms’ opportunity costs. I then conduct simulation exercises to explore differences with respect to the average time it takes for investment to take place and the resulting market structure. The last section presents some concluding remarks.

METHODS

Traditional methods of investment planning like the net present value (NPV) suggest that a project should be executed if the net present discounted value of its future expected cash flows is positive. Probably due to its tractability, NPV is a widely used method in economics, finance, and accounting. A common drawback of this methodology, however, is that “just because an initiative has a positive NPV or provides an optimal return on investment does not mean that is the best usage of the funding” (Damodaran, 1999). That is, a positive NPV should not be a sufficient condition as it should be contrasted with the other alternatives that investors have. Real options analysis (ROA), on the other hand, is a valuation methodology that accounts for the value of options, such as the possibilities of expansion, contraction, altering the project along the way, building in stages, postponing, cancelling, etc.

The analysis I conduct in this paper is a mix of both techniques. For instance, this model employs the NFV technique in order to provide a necessary (but not sufficient) condition for executing an investment project. Given that investors evaluate the possibility of executing an irreversible all-or-nothing project, investment in stages, an ROA element, cannot be applied. However, I do incorporate valuable ROA elements as sufficient conditions for determining the optimal timing for investing. First, the value of postponing: a project may have a positive net
present value but still not be accepted right away, because the firm may gain by accepting the project in a future period. Second, the value of waiting: entrepreneurs may delay in order to obtain more information and thus reduce the risk.

To sum up, this model uses the NPV method only as a necessary condition for investing, while it applies some pertinent ROA elements into the model in order to provide sufficient conditions and, thus, to make the analysis more realistic and complete.

THE MODEL

Consider a simple model of two firms that produce the same good in an industry whose demand (and thus price) remains uncertain until at least one of the firms is actually in the market selling it. In addition, suppose for simplicity that demand for this product exhausts after two periods of activity. Therefore, the second firm still has the chance to enter the market once uncertainty has been resolved after the first firm has decided to invest. This assumption should not be crucial for the results when compared with demand that lasts arbitrarily longer.

Investment is irreversible and involves a fixed sunk cost \( I_0 \), which is constant across firms and over time. Projects take one period to finish, thus once a firm decides to invest, it becomes operational at the beginning of the following period. Figure 1 summarizes the timing of events that follow the first investment decision.

Let us consider the case in which only one firm decides to initially invest. Without loss of generality, let us refer to such a firm as "firm 1," and let \( t_0 \) be the period that it decides to start the investment. Then, in period \( t_0 + 1 \), firm 1 starts selling in the market and firm 2 makes its investment decision, conditional on the certain realization of true demand, which

Figure 1: Timing of investment decisions

- Investment decisions
- Firm 1 starts selling
- Firm 2 starts selling (if H)
- Firm 1 starts investing
- Firm 2's investment decision
- If L, both firms exit the market
- Demand is exhausted (if H)

\[ t_0 \quad t_0+1 \quad t_0+2 \quad t_0+3 \]

- Firm 1's investment process
- Firm 1 sells alone in the market
- Both firms sell in the market
- Firm 2's investment process (if applies)

Can be either high (H) or low (L). If demand turns out to be high, firm 2 starts investing and both firms sell in the market in period \( t_0 + 2 \). In period \( t_0 + 3 \) the demand is exhausted. If demand is low, then firm 2 does not invest and firm 1 stops selling. 4

Firms form their expectations on the state of demand through both the arrival of signals, which can either be positive (+) or negative (−), and by observing the action (or inaction) taken by the other firm. In particular, firms hold an initial belief \( (\pi_0) \) that demand will be high and they update it each period upon receiving the signals. Therefore, everything else constant, the probability that the state of demand is high (low) perceived by firms depends positively on the
number of good (bad) signals received. The true state of demand is revealed once the project is running.

Both firms are risk-neutral and maximize expected profits. In addition, assume that firms hold a reserve utility \( r_0 \) equal to the expected present discounted value of profits from the next best alternative investment available to them. Assume that both firms have the same reserve utility in order to isolate the effects of asymmetry in information only on the state of demand. This assumption is relaxed later in the paper.

The equilibrium price and quantity depend on the resulting market structure. That is, unlike in Chamley and Gale (1999), firms also have the incentive to be “first” in the market. Namely, a firm can have temporary monopolistic power if it sells alone, while firms would have to share the market if both operate at the same time. For simplicity, I assume that in such a case, firms would behave as duopolists, and price and quantities are set as in a Cournot solution.

Therefore, each firm decides whether to invest (“in”) or not (“out”) in period \( t_0 \). Their payoffs will depend on their own and the other firm’s decisions, and on the actual state of demand. Figure 2 summarizes the payoff structure in period \( t_0 + 1 \) if demand turns out to be low. Basically, the payoffs are \(-I_0\) if the firm decides to invest, and 0 otherwise. The game ends after period at \( t_0 + 1 \).

**Figure 2: Payoffs in period \( t_0 + 1 \) (demand L)**

<table>
<thead>
<tr>
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<th>IN</th>
<th>OUT</th>
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<tbody>
<tr>
<td>IN</td>
<td>(-I_0, -I_0)</td>
<td>(-I_0, 0)</td>
</tr>
<tr>
<td>OUT</td>
<td>(0, -I_0)</td>
<td>---</td>
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</tbody>
</table>

On the other hand, if demand is high, then the payoff structure in period \( t_0 + 1 \) is shown in Figure 3.

**Figure 3: Payoffs in period \( t_0 + 1 \) (demand H)**

<table>
<thead>
<tr>
<th></th>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN</td>
<td>(\pi_{\text{Cournot}} - I_0, \pi_{\text{Cournot}} - I_0)</td>
<td>(\pi_{\text{Monopolistic}} - I_0, 0)</td>
</tr>
<tr>
<td>OUT</td>
<td>(0, \pi_{\text{Monopolistic}} - I_0)</td>
<td>---</td>
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</tbody>
</table>

If demand is revealed to be high, then in period \( t_0 + 2 \), both firms continue to share the market in a Cournot solution if both firms invest together in period \( t_0 \), or produce as in a Stackelberg solution if only one firm decides to start the investment at \( t_0 \), resulting in higher profits for the “leader” (firm 1) than for the “follower” (firm 2). The game ends after period \( t_0 + 2 \). Payoffs are summarized in Figure 4.
Figure 4: Payoffs in period $t_0 + 2$ (demand H)

<table>
<thead>
<tr>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{Cournot} - l_0$, $\pi_{Cournot} - l_0$</td>
<td>$\pi_{Leader} - l_0$, $\pi_{follower} - l_0$</td>
</tr>
<tr>
<td>$\pi_{follower} - l_0$, $\pi_{Leader} - l_0$</td>
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</tbody>
</table>

In order to compute prices, quantities and profits, let us assume a simple linear demand function $p = a - b \cdot q$, where $b$ is a positive parameter that is fixed and known to both investors. Parameter $a$, on the other hand, is ex ante unknown and captures the state of demand. Namely, demand can be high when $a = a_H$ with probability $\pi$, or low and $a = 0$ with probability $1 - \pi$. Under a low state of demand, the price level would be zero for any quantity, and firms will not find it profitable to invest. Suppose also for simplicity that production involves zero operating costs.

Therefore, firms maximize expected present discounted value of profits:

$$\max_{\pi_{t+1}} \mathbb{E}_t (\Pi_t) = \pi_{t+1} \left\{ \beta \cdot p_{t+1}(Q_{t+1}) \cdot q_{t+1} + \beta^2 \cdot p_{t+2}(Q_{t+2}) \cdot q_{t+2} \right\} - l_{t+1}$$ (1)

where $\beta \in [0,1]$ is the factor through which firms discount future revenues, and $Q = q^1 + q^2$.

Firm $i$'s participation constraint as of the period of the investment is given by:

$$\pi_{t+1} \left\{ \beta \cdot p_{t+1}(Q^*_t) \cdot q_{t+1}^*(E(q_{t+1}^*)) + \beta^2 \cdot p_{t+2}(Q^*_t) \cdot q_{t+2}^*(E(q_{t+2}^*)) \right\} \geq I_t + r_0$$ (2)

where $q_{t+1}^*(E(q_{t+1}^*))$ represents the optimal quantity response by firm $i$ given an expected strategy $q_{t+1}^*$ from the other firm, and $Q^* = q^1(E(q_{t+1}^*)) + E(q_{t+2}^*)$ is the industry's total expected production.

As mentioned earlier, the equilibrium price level $p_{t+1}$ depends on the market structure resulting from investment decisions in previous periods. Specifically, if a firm produces alone in the market, then the resulting price and quantities are the standard monopoly solution, i.e., $p_{t+1} = a_H^2/2$ and $q_{t+1} = a_H^3/2b$, with implied revenues (and operational profits) $\Pi_{t+1} = (a_H^2)^2/4b$. If both firms invest in the same period, the result is a standard Cournot solution, where price, quantity produced and revenues are $p_{t+1} = a_H^3/3$, $q_{t+1} = a_H^3/3b$ and $\Pi_{t+1} = (a_H^3)^2/9b$, respectively. Finally, if only one firm operates in the first period (firm 1), and the demand turns out to be high, then firm 2 enters the market with probability one, observe firm 1's outcome and act as a follower on a Stackelberg game. Consequently, the price drops in the second period to $p_{t+2} = a_H^4/4$, firms 1 and 2 produce, $q_{t+1}^* = a_H^3/2b$ and $q_{t+2}^* = a_H^4/4b$ and receive per-period revenues of $\Pi_{t+2} = (a_H^2)^2/8b$ and $\Pi_{t+3} = (a_H^3)^2/16b$, respectively.

Note that the Stackelberg two-period expected profits for the firm that first enters the market is:

$$\pi_{t+1} \cdot \beta \left( 1 + \frac{\beta}{2} \right) \cdot \left( \frac{a_H^2}{4b} \right) - l_{t+1}$$ (3)

while if both firms start their investments in the same period, then the two-period Cournot's expected profits are:
\[ \pi_t \cdot \beta \cdot (1 + \beta) \frac{(a^H)^2}{9b} = I_t \]

(4)

Consequently, as of period \( t \) (when neither firm has made investment decisions yet), the necessary condition for a firm \( i \) to decide to invest is such that the 2-period present discounted value of the flow of profits is not lower than the investor’s opportunity cost of funds:

\[ \pi_{i,i} \cdot \gamma^{-i} \cdot \beta(1 + \frac{\beta}{2}) \frac{(a^H)^2}{4b} + \pi_{i,j} \cdot (1 - \gamma^{-i}) \cdot \beta(1 + \beta) \frac{(a^H)^2}{9b} - I_t \geq r_0 \]

(5)

where \( \gamma^{-i} \) is the probability that the other firm finds it unprofitable to invest in period \( t \). In addition, parameters \( a^H, b, \beta, I_t \) and \( r_0 \) must satisfy that if \( \pi_t = 1 \), then profits from equations (3) and (4) are greater than \( r_0 \) for both firms.

Note that equation (5) is not a sufficient condition for a firm to start investing in period \( t \) since it does not consider the option value, i.e., the value that a firm gives to waiting and hoping that the next period will bring more convincing information on the future state of demand. In particular, a firm may want to wait until the period after the first firm starts investing in order to have certainty on the state of demand, even if that decision involves giving up one period of potential profits as a Cournot duopolist. That is, investment decisions are more likely to be postponed if investors believe that new information will soon become public.

Consequently, an ex-ante sufficient condition for a firm to invest in period \( t \) would take the form:

\[ \gamma^{-i} \left( \pi_{i,i} \cdot \beta(1 + \frac{\beta}{2}) \frac{(a^H)^2}{4b} + \pi_{i,j} \cdot (1 - \gamma^{-i}) \cdot \beta(1 + \beta) \frac{(a^H)^2}{9b} - (I_t + r_0) \geq \right. \]

\[ \left. (1 - \gamma^{-i}) \cdot \pi_{j,i} \left( \beta^2 \frac{(a^H)^2}{16b} - (\beta I_t + r_0) \right) \right) \]

(6)

where the left-hand side of equation (6) captures the expected profits (net from the opportunity cost of funds) from investing in period \( t \), while the right-hand side represents the expected net profits from waiting one period and observing true demand following firm 2’s investment.

Put differently, investors are faced each period with a trade-off between investing that period and potentially earning two periods of profits if demand is high (captured by the left-hand-side of equation 6), and waiting one period and taking advantage of more information and earning one period of potential profits, particularly if the firm sells alone. Note that the above condition is different from condition (5) in that it contains a second term on the right hand side. This term captures the concept of option value or the value of waiting, which is the extra value that firms require on top of their reserve utility in order to start a new investment in period \( t \).

Note also that the term \( (\beta I_t + r_0) \) on the right-hand side of inequality (6) is multiplied by \( \pi_t \), which reflects the fact that the firm has the option of incurring no costs if demand turns out to be low. Consequently, the expected present discounted value of staying uncommitted in period \( t \) and waiting for certain information in period \( t + 1 \) is captured by the term \( (1 - \gamma^{-}) \cdot (1 - \pi_t) \cdot (\beta I_t + r_0) \), or the expected present discounted value of savings from not losing the investment cost and reserve utility if demand is low.
Figure 5: Signals' conditional probability of occurrence

<table>
<thead>
<tr>
<th>Belief</th>
<th>Demand</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_0 )</td>
<td>( H )</td>
<td>Positive (+)</td>
</tr>
<tr>
<td>1 - ( \pi_0 )</td>
<td>( L )</td>
<td>Negative (-)</td>
</tr>
</tbody>
</table>

How do the firms model their beliefs?

Assume that the initial belief that demand will be high (\( \pi_0 \)) is exogenous and equal for both firms. Firms then update their beliefs based upon the signals they receive at the end of each period. Figure 5 presents the probability of occurrence for signals, given the true (though still unrevealed) state of demand.

If the true state of demand is high (\( H \)), investors will receive a positive signal with probability \( u \in [0, 1] \), and a negative signal with probability \( 1 - u \). Similarly, if the true state of demand is low (\( L \)), firms will receive a positive signal with probability \( v \in [0, 0.5] \), and a negative signal with probability \( 1 - v \). Assume that \( u \) and \( v \) are exogenous and commonly known, and that investors receive these signals independently and privately over time.

Consequently, upon receiving a positive signal at the end of each period, both firms update their beliefs through Bayes' rule. That is:

\[
\pi_{n+1}^+ = \text{prob}_n(H|+) = \frac{\text{prob}_n(H) \cdot \text{prob}(+|H)}{\text{prob}_n(H) \cdot \text{prob}(+|H) + \text{prob}_n(L) \cdot \text{prob}(+|L)}
\]  

or equivalently,

\[
\pi_{n+1}^+ = \frac{\pi_n \cdot u}{\pi_n \cdot u + (1 - \pi_n) \cdot v}
\]

Then, probability \( \pi_{n+1} \) will be higher the greater the prior belief \( \pi_0 \), the greater probability \( u \) is (that is, the probability of receiving a good signal if demand is high) and the lower \( v \) is (i.e., the probability of receiving a false good signal). Note that upon receiving a positive signal, it is true that \( \pi_{n+1} \geq \pi_n \). That is, investors are more likely to start investments upon receiving positive signals. Note that the evolution of \( \pi_n \) over time determines the value of waiting. That is, given that the higher the value of \( u \) and the lower the value of \( v \), the more information about the true state of demand is revealed, firms have the incentive to wait for stronger information about demand before deciding to invest.
Similarly, upon receiving a negative signal:

\[
\pi_{i,t}^{-} = \frac{\pi_{i}(1-u)}{\pi_{i}(1-u) + (1-\pi_{i})(1-v)}
\]

which implies that \(\pi_{i,t}^{-} \leq \pi_{i}\).

**THE BENCHMARK CASE: COMPLETE INFORMATION**

Let us first discuss the benchmark case of asymmetric information. In this framework, signals that firms receive every period are common knowledge. Therefore, investors observe the same signals as both firms receive and, consequently, they update their beliefs in the same way. That is, the game is symmetric and:

\[
\pi_{i,t} = \pi_{2,t} = \pi_{i}, \quad \forall t
\]

(10)

Naturally, under perfect information, a firm knows the other firm's investment decision in any period, considering all the information captured up until then. Therefore, symmetric information implies neither a monopolist nor a Stackelberg solution. As a consequence, the first term on the right-hand side and the entire left-hand side of equation (6) drop out (also \(\gamma^{d} = 0\)) and the necessary participation constraint for both firms under a Cournot solution is:

\[
\pi_{i} \cdot \beta(1 + \beta) \left(\frac{a^{\mu}}{gb}\right)^{2} - I_{o} \geq r_{o}
\]

(11)

In principle, the investment condition above may not necessarily be a sufficient condition since it must be compared to the expected present discounted value of profits if a firm decides to wait, hoping that new information will reduce uncertainty. Therefore, a sufficient condition is such that \(\beta \mathbb{E} \Pi_{s} > \mathbb{E} \Pi_{l}\) for all \(s \in [1, \infty)\). That is, waiting additional periods would imply an expected present discounted value of profits being no greater than the ones expected for the current period.

\[
\pi_{i}^{s} \cdot \beta(1 + \beta) \left(\frac{a^{\mu}}{gb}\right)^{2} - \beta I_{o} \leq \left(\pi_{i} \cdot \beta(1 + \beta) \left(\frac{a^{\mu}}{gb}\right)^{2} - I_{o}\right)
\]

(12)

**PROPOSITION.** Under symmetric information, a firm whose Cournot solution participation constraint (equation 11) is satisfied would not postpone investment for future periods.

**PROOF.** Both firms would wait at least one more period when \(\beta \mathbb{E} \Pi_{s} > \Pi_{l}\), which is positive only when \(\beta \mathbb{E} \Pi_{s} > \pi_{i}\).

Note that since the next signal is unknown as of the current period, then

\[
\pi_{i,t}^{+} = \text{prob}(+) \cdot \pi_{i,t}^{+} + [1 - \text{prob}(+)] \cdot \pi_{i,t}^{-}
\]

(13)
where the probability of receiving a positive signal is \( \text{prob}(+) = q \cdot \pi + s(1 - \pi) \). Using Bayes' rule (equations 8 and 9), it follows that

\[
\pi_{t+1}^* = \pi_t
\]

That is, a firm's best guess about next period's \( \pi \) is the current \( \pi \). Then, equation (12) implies the following condition

\[
\pi_t \cdot \beta(1 + \beta) \left[ \frac{(a^n)^2}{9b} - I_s \right] \geq 0
\]

which is positive following the participation constraint from equation (11). Therefore, firms will not postpone investment for one period.

In addition, since the result in equation (14) applies for any future period, firms will not consider postponing investment for any period beyond \( t+1 \).

THE ASYMMETRIC INFORMATION CASE

Under asymmetric information, signals are not common knowledge and so firms' beliefs may evolve differently over time. Therefore, given that there is uncertainty about the other firm's investment decision, action or inaction from one firm constitutes valuable information for the other.

Note that a firm \( i \)'s sufficient condition for investment in period \( t \) (equation 6) is \textit{ex-ante} symmetric. Following the result from equation (14), and due to asymmetries in information, each firm's best conjecture about the other firm's \( \pi \) is \( \pi_0 \). Equation (6) is then transformed to:

\[
\gamma \cdot \left[ \pi_0 \cdot \beta(1 + \beta) \left( \frac{(a^n)^2}{2} \right) + (1 - \gamma) \cdot \beta(1 + \beta) \cdot \frac{(a^n)^2}{9b} - (I_s + r_s) \right] \geq 0
\]

Equation (16) represents firms' \textit{ex-ante} investment participation constraint. Note that this condition is symmetric in that it implies that the probability of participation is the same for both firms. That is, \( \gamma^1 = \gamma^2 = \gamma \). Therefore, since \( \gamma \) represents the probability that neither firm is investing, and since the game only makes sense when at least one firm invests, it follows that both firms take \( \gamma = 0 \) and, thus, each firm assumes \textit{ex-ante} that the other firm will invest as well. Therefore, the sufficient condition for a firm \( i \) to also invest in period \( t \) is:

\[
\pi_{it} \cdot \beta(1 + \beta) \cdot \frac{(a^n)^2}{9b} - (I_s + r_s) \geq \pi_{it} \left[ \beta^2 \cdot \frac{(a^n)^2}{16b} - (\beta I_s + r_s) \right]
\]

(17)
That is, the expected two-period duopolist discounted revenues, net of the certain (and sunk) investment cost and reserve utility \( (l_i + r_0) \), are greater than the one-period Stackelberg follower’s net expected discounted profits.

Note that the result from the previous proposition also applies to the asymmetric information scenario. That is, although firms may postpone investment in order to obtain certain information on demand from the action of the other firm, they will not postpone investment with the objective of reducing uncertainty by receiving more signals in future periods.

Therefore, a firm will choose to invest in period \( t \) when its \( \pi_t \) reaches a cutoff \( \pi^* \); that is, when

\[
\pi_t \geq \frac{\beta \cdot \left( a^2 \right)^{2} \left( 16 + 7 \beta \right)^{2}}{9b^{2} \left( 144 + \beta l_i + r_0 \right)} = \pi^*
\]

The impacts of parameter changes to the investment cutoff in equation (16) are unambiguous and straightforward. First, increases in the state of demand \( (a^2) \) and reductions in the demand slope parameter \( (b) \) increase the probability of investing today (i.e., they reduce \( \pi^* \)). Though a higher \( a^2 \) or a lower \( b \) raise the expected revenues both today and in the next period, the impact on investing in the current period is stronger, as it affects expected revenues for two periods instead of just one. Second, the discount factor \( (\beta) \) positively impacts investment in the current period. Though investors are more patient and the value of waiting goes up, the present discounted value of investing today also (and more strongly) increases.

Finally, the greater the value of the initial investment \( (l_i) \) and the higher the value of the next best alternative that investors have \( (r_0) \), the less likely it is for the investment to be undertaken, as the present discounted value of the project (net of the reserve utility) falls. Though increases in investment also reduce the attractiveness of investing in the second period, the net effect is positive towards not investing in the current period, given that investment (and its opportunity cost) can be avoided in the event that demand turns out to be low.

**Firm 2’s investment decisions**

Recall that we refer to firm 2 as the firm that makes its investment decision after observing firm 1’s decision. Naturally, if the investment condition (equation 18) is satisfied for both firms, then both firms make their investment decisions simultaneously. However, if the investment condition is satisfied for firm 1 but not firm 2, then firm 1 invests while firms 2 waits. Note that the concept of waiting does not only mean that firm 2 would remain inactive in period \( t \) to then contemplate investing in period \( t + 1 \). Since the decision of inaction is reversible, firm 2 could and would re-evaluate its initial choice given the fact that firm 1’s action is providing more information to firm 2. In particular, given that both firms have the same reserve utility \( (r_0) \), firm 2 knows exactly the net number of positive signals over negative signals that firm 1 received. Consequently, firm 2 has the advantage over firm 1 of having complete information about the likely state of demand before making its investment decision.

Let

\[
\pi^*_2 > \pi^*_1
\]

be the updated belief held by firm 2 about the state of demand after observing firm 1’s action. Then, if \( \pi^*_2 < \pi^*_1 < \pi^*_3 \), firm 2 would invest and firm 1’s action would be crucial for firm 2’s decision in period \( t \). However, the acquisition of more positive signals from firm 1 does not guarantee that firm 2 will invest, as it could have received enough negative signals so that the investment condition is still not satisfied. That is, firm 2 will not invest if \( \pi^*_2 < \pi^*_3 < \pi^*_1 \). In this case, note that firm 1 also gets to know that, given that firm 2 has complete information about signals, ex-post, not investing would have been a better decision.
However, unlike the choice to wait, investment decisions are irreversible. Note that since the game is symmetric and firm 1’s best guess about firm 2’s $\pi_{1,2}$ is $\pi_0$, firm 1’s best guess is that firm 2 will also invest after observing its action. Therefore, unlike in Chanley and Gale (1994), firm 1 has no ex-ante incentive to be “last” and obtain more information from the other firm, even if firm 2 receives ex-post benefits from being last.

The case of heterogeneity in preferences

So far we have assumed that the firms’ reserve utilities are the same, which seems to be a priori a reasonable assumption. However, it is also reasonable to assume that due to, say, differences in their ownership structure, firms may take a different approach to risk and have different opportunity costs of funds. Pardo (2012) points out that even though entrepreneurs are commonly modeled in the literature as risk-neutral agents for the sake of simplicity, risk aversion could be a more realistic assumption for private entrepreneurs. As Gale and Hellwig (1985) put it, risk neutrality among larger investors “can be justified as a consequence of risk-pooling.” However, as Moskowitz and Vissing-Jørgensen (2002) find, privately-owned companies are typically small and owned by few or just one entrepreneur. They also show that private entrepreneurs usually invest at least 50 percent of their assets in a single private company. Therefore, smaller private entrepreneurs are less likely to have access to complete risk-pooling for their idiosyncratic risks, and thus tend to be more vulnerable to project-specific, uninsurable risk.

Pardo (2012) shows that risk-averse entrepreneurs require a private equity premium, or a premium that entrepreneurs demand due to the stochastic nature of their investment returns. Although not explicitly modeled in this paper, an extra risk premium for investment can be interpreted as an investor requiring an expected present discounted value of investing to be higher for risk-averse than for the case of risk-neutral entrepreneurs. Therefore, differences in reserve utility among firms could also be understood as differences in the degree of risk aversion, since some would require greater expected present discounted value of profits in order to remain indifferent between investing or not in each period.

In addition, differences in $r_0$ could be interpreted as differences in optimism about variables that investors may consider to be relevant to their decision-making. An increase in $r_0$ could be equivalent to an increase in optimism, and so it could positively affect the probability of undertaking the firm’s investment project. Therefore, periods of optimism may lead to earlier investment schedules, which could be expected to take place during economic booms. In that case, firms would tend to be less cautious about operating in niches where they could obtain higher profits. This phenomenon is also consistent with previous literature. Pardo (2012) finds that a counter-cyclical private equity premium can magnify the impact of shocks over time. Shocks that lead to reductions in entrepreneurial wealth would increase entrepreneurs’ effective level of risk aversion. Consequently, a higher private equity premium would increase (equivalently in this section, $r_0$ would increase) and, as a response, entrepreneurs would optimally cut investment and therefore, in general equilibrium, aggregate production.

If we assume that firms differ only in their reserve utility (i.e., $r_{0,1} \neq r_{0,2}$), and these parameters are common knowledge, then the game is no longer symmetric, though the market structure would again depend on the receipt of signals and the parameters of the model. In general, if firm $i$ decides to invest, since the $r_0$’s are known, it will again provide the other firm (-$i$) with clear information about all the signals received by both firms. Firm -$i$ will have the benefit of receiving more signals and thus a better estimate of the true state of demand. Therefore, the general rule $\gamma^T_{r_{0,i}}$ remains simple. Given firm $i$’s decision to invest in period $t$, the probability of not investing for firm -$i$ is
\[ \gamma^* = \begin{cases} 
  0 & \text{if } \pi^*_i < \pi_c^* \\
  1 & \text{if } \pi^*_i \geq \pi_c^* 
\end{cases} \] (19)

where \( \pi_c^* \) is the cutoff value of the probability of high demand for firm \(-i\) to also invest.

In particular, given that the \( r_0 \)'s are known and that each firm's best guess about the other's current \( \pi \) is its initial \( \pi \) (i.e., \( \pi_0 \)), if without loss of generality we assume that \( r_{01} < r_{02} \), then if firm 1 decides to invest, then in expectations the net number of positive signals it reveals will not be enough for firm 2 to invest (due to firm 2's higher reserve utility) and thus \( \gamma^1 = 1 \). In contrast, if firm 2 invests, then in expectations the number of positive signals reveals will be enough for firm 1 to invest. Thus, \( \gamma^2 = 0 \). Therefore, the critical values for the probability of high demand for each firm to invest are:

\[ \pi^*_1 = \frac{J_i + r_{01}}{\beta (1 + \frac{\beta}{2}) \frac{(a^h)^2}{4b}} \] (20)

and

\[ \pi^*_2 = \frac{J_i + r_{02}}{\beta \left( \frac{(a^h)^2}{9b} \left[ \frac{16 + 7\beta}{144} \right] + b \beta + r_{03} \right)} \] (21)

If the \( r_0 \)'s are unknown, on the other hand, then a firm's decision to invest does not provide clear information about the actual positive signals received until then, but about the expected number of positive signals received by it. In particular, if firms have the same expected reserve utility (say, \( r_0 \)), then the game is again ex-ante symmetric and equivalent to the case of equal \( r_0 \)'s. Similarly, if the reserve utilities are unknown and the expected values differ between firms (again due to differences in their ownership structure), then the resulting game is once more asymmetric and equivalent to the case of different, but known, \( r_0 \)'s. Even though individual responses may significantly vary as a consequence of introducing unknown \( r_0 \)'s, in the aggregate by the law of large numbers, these differences may evaporate.

**SIMULATIONS**

This section presents some simulation exercises. The objective is to examine the differences in terms of the amount of time to invest and market structure that results from the different cases analyzed in the theoretical model.

I conduct a Monte Carlo simulation by randomly generating the signals that firms receive each period from which they form their beliefs with respect to the expected state of demand. Positive signals arrive with probability \( \pi \) if the actual state of demand is high and with probability \( 1 - \pi \) if demand is low. I then follow the firms' responses in terms of their probability updates (\( \pi_0 \)) and investment decisions for each of the following cases:

- **Case 1:** symmetric information, same reserve utility (\( r_0 \))
- **Case 2:** asymmetric information, same \( r_0 \)'s
- **Case 3:** different \( r_0 \)'s, observed
Case 4: different \( r_0 \)'s, unobserved, same mean  
Case 5: different \( r_0 \)'s, unobserved, different mean

I execute 50,000 replications of the experiment for each case, and then aggregate the responses and observe the average number of periods it takes for the first firm to invest as well as the response from the other firm.

Initially, I assume the following parameter values (base scenario): initial belief \( (\tau_0) = 0.1; \) discount factor \( (\beta) = 1/1.05; \) demand in high states \( (d^H) = 5; \) price elasticity of demand parameter \( (b) = 0.05; \) initial investment \( (I) = 25; \) reserve utility \( (r_0) = 5\% \) of investment; low reserve utility \( (r_0,l) = 3\% \) of investment; high reserve utility \( (r_0,h) = 7\% \) of investment; probability of high demand (if actual demand is high) \( (u) = 0.6; \) and probability of high demand (if actual demand is low) \( (v) = 1 - u. \) I also conduct a sensitivity analysis to this parameter selection. Simulation results are presented in table 1.

In case 1, both firms always invest at the same time as both signals are observed simultaneously every period. In the asymmetric information case, each firm processes less information about the actual state of demand and, consequently, investment postponement is considerably longer than in case 1. Case 3 is also a case of asymmetric information but, unlike case 2, firms differ in their reserve utilities \( (r_0). \) Therefore, while equal information sharing takes place in case 2 regardless of which firm invest first (therefore a two-firm outcome is more likely), in case 3, the firm with the lower reserve utility (which tends to invest first) shares less information about the state of demand than when the firm with the highest reserve utility takes first. Consequently, two firms investing simultaneously is a considerably less frequent solution than in case 2.

There is a slight increase in the outcome of both firms investing when the \( r_0 \)'s are unknown (case 4) relative to when they are known (case 2). In case 4, firms expect ex-ante that the other firm has the average \( r_0 \) regardless of its actual one. Thus, there will be discrepancies between the number of signals transmitted and that which is received. That is, as investors are uncertain about the others' reserve utility, under some circumstances (in this case, relatively higher expected profits due to high \( d^H \) and/or low \( I)), \) firms are more likely to follow others, consistent with "herd behavior" in investment. In case 5, however, results are very similar to case 3. It seems that the ex-ante differences in reserve utility influence the resulting market structure more strongly than the added uncertainty does.

Naturally, the resulting time to invest and market structure in all five cases are sensitive to the parameter selection. For instance, the more information signals provide about the actual state of demand \( (u), \) the more quickly firms find it profitable to invest. In terms of the market structure, however, results are less clear. On one hand, firm may tend to act more quickly as signals provide more information (which makes the one-firm solution more likely). On the other hand, improvements in the quality of signals imply stronger information sharing (which makes the two-firm solution more likely). Neither of these two conflicting forces appears to dominate uniformly as \( u \) increases.

Similarly, as expected, increases in demand in high states \( (d^H) \) and reductions in investment amounts \( (I) \) tend to accelerate the timing of investment. In addition, both higher \( d^H \) and lower \( I \) raise the expected investment profitability for both firms, thus making the two-firm solution more likely to occur in cases 2 and 3. Higher \( d^H \) and/or lower \( I, \) however, causes investment to take place sooner, increasing the likelihood that any firm in particular could invest before the other, and increasing the likelihood of a one-firm solution. It appears that neither effect offsets the other in case 4.
Table 1: Simulation results and sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>$u = 0.55$</th>
<th>$u = 0.7$</th>
<th>$d^H = 3.5$</th>
<th>$d^H = 6$</th>
<th>$I = 15$</th>
<th>$I = 35$</th>
<th>$\pi_0 = .2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of periods to invest</td>
<td>9.96</td>
<td>40.04</td>
<td>2.50</td>
<td>14.99</td>
<td>4.93</td>
<td>5.02</td>
<td>14.93</td>
<td>4.99</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of periods to invest</td>
<td>32.74</td>
<td>122.08</td>
<td>8.54</td>
<td>36.79</td>
<td>24.96</td>
<td>24.89</td>
<td>36.75</td>
<td>25.09</td>
</tr>
<tr>
<td>one firm invest (%)</td>
<td>24.8%</td>
<td>22.7%</td>
<td>17.5%</td>
<td>18.0%</td>
<td>32.9%</td>
<td>32.6%</td>
<td>18.1%</td>
<td>31.4%</td>
</tr>
<tr>
<td>both firms invest (%)</td>
<td>75.2%</td>
<td>77.3%</td>
<td>82.6%</td>
<td>82.0%</td>
<td>67.1%</td>
<td>67.4%</td>
<td>81.9%</td>
<td>68.6%</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
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<td></td>
</tr>
<tr>
<td># of periods to invest</td>
<td>8.67</td>
<td>34.31</td>
<td>2.23</td>
<td>16.89</td>
<td>4.21</td>
<td>4.24</td>
<td>12.85</td>
<td>4.25</td>
</tr>
<tr>
<td>one firm invest (%)</td>
<td>91.2%</td>
<td>91.4%</td>
<td>94.0%</td>
<td>79.1%</td>
<td>95.1%</td>
<td>95.1%</td>
<td>88.4%</td>
<td>95.1%</td>
</tr>
<tr>
<td>both firms invest (%)</td>
<td>8.8%</td>
<td>8.6%</td>
<td>6.0%</td>
<td>20.9%</td>
<td>4.9%</td>
<td>4.9%</td>
<td>11.6%</td>
<td>4.9%</td>
</tr>
<tr>
<td><strong>Case 4</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of periods to invest</td>
<td>32.84</td>
<td>122.83</td>
<td>8.52</td>
<td>37.03</td>
<td>24.97</td>
<td>25.11</td>
<td>36.71</td>
<td>25.12</td>
</tr>
<tr>
<td>one firm invest (%)</td>
<td>15.1%</td>
<td>22.5%</td>
<td>17.4%</td>
<td>17.8%</td>
<td>18.9%</td>
<td>19.6%</td>
<td>18.5%</td>
<td>18.9%</td>
</tr>
<tr>
<td>both firms invest (%)</td>
<td>84.9%</td>
<td>77.5%</td>
<td>82.6%</td>
<td>82.2%</td>
<td>81.1%</td>
<td>80.4%</td>
<td>81.5%</td>
<td>81.1%</td>
</tr>
<tr>
<td><strong>Case 5</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of periods to invest</td>
<td>8.70</td>
<td>33.97</td>
<td>2.23</td>
<td>16.83</td>
<td>4.15</td>
<td>4.21</td>
<td>12.84</td>
<td>4.26</td>
</tr>
<tr>
<td>one firm invest (%)</td>
<td>91.3%</td>
<td>91.9%</td>
<td>94.1%</td>
<td>79.8%</td>
<td>95.6%</td>
<td>95.5%</td>
<td>88.9%</td>
<td>95.4%</td>
</tr>
<tr>
<td>both firms invest (%)</td>
<td>8.7%</td>
<td>8.1%</td>
<td>5.9%</td>
<td>20.2%</td>
<td>4.4%</td>
<td>4.5%</td>
<td>11.1%</td>
<td>4.6%</td>
</tr>
</tbody>
</table>

15
Finally, the higher the initial belief about future high demand ($\pi_0$), the faster investment takes place. In addition, as any particular firm reaches its probability cutoff ($\pi^*$) more quickly, the one-firm outcome becomes more likely to occur across all five cases.

CONCLUSIONS

This paper explores some factors behind investment decisions in the context of asymmetries in information. Firms’ actions are ultimately determined by private signals that they independently receive and through which they build on their beliefs about the future state of demand. That is, the timing of investment choices depends critically on how fast uncertainty is resolved over time, and the resulting market structure depends on how important the ex-post asymmetry in information is.

In the benchmark case of complete information, where both firms build their beliefs equally, firms act together as soon as it is optimal to invest. In contrast, when asymmetry in information is introduced, important differences in market structure may result. In particular, the timing of investment could change in two opposing ways. On the one hand, the greater the perceived probability that the other firm will invest in the current period, the greater the competition effect and, thus, the sooner a firm would tend to invest, including the possibility of generating suboptimal decisions (i.e., investment earlier than in the benchmark case). On the other hand, information “noise” may cause the value of waiting to gain importance in the optimal timing of investment, as firms may choose to postpone investment in order to collect more information, both from signals and from actions by others.

I also examine in this paper the effects of unobservable differences in preferences, as captured by each firm’s reserve utility, on investment decisions, the timing of investment and the resulting market structure. Due to differences in ownership structure, firms may hold a distinct approach towards risk and thus have different opportunity costs of funds. For instance, risk aversion could be a more realistic assumption for private entrepreneurs due to their more constrained access to complete risk-pooling for their idiosyncratic risks. Though not explicitly modeled in this paper, the private equity premium can be interpreted as an investor requiring a higher reserve utility given his higher vulnerability to project-specific, uninsurable risk. Changes in reserve requirements can affect the probability of undertaking the firm’s investment project. In particular, when a firm’s reserve utility is unknown, one firm’s decision to invest does not provide clear information about the actual signals received, rather only about the expected number of signals received. Under some circumstances, uncertainty about each other’s preferences may make “herd behavior” in investment more likely due to the discrepancy between signals transmitted and signals received.

Lastly, I conduct simulation of the model by randomly generating the signals that firms receive each period, following the probability updates by firms, and then aggregating the responses over 50,000 replications of the experiment for each case analyzed in the theoretical model. Simulation exercises tend to confirm the models’ predictions regarding time to investment and the resulting market structure for the different cases.

To sum up, incomplete information and/or information asymmetries may lead to suboptimal solutions, including postponing, not taking advantage of profitable investment opportunities or, conversely, embarking upon projects earlier than optimal. Therefore, in terms of policy implications, improvements in information technology and transparency could alleviate the welfare costs imposed by information problems, which are particularly important in times when an economy tries to recover from a recession.
ENDNOTES

1 Prasad, Agenor & McDermott (1999), Backus & Kehoe (1992), among many others, provide empirical evidence supporting much higher average output volatility in emerging economies than in industrialized economies.

2 Some examples are Yahoo among search engines, Amazon.com among booksellers, CDNow among music stores.

3 Examples of failures are much more abundant than those of success, but some notable examples include Excite.com, Pets.com, latminute.com, etc.

4 The special case that both firms invest in period t can be captured in Figure 1 as if both firms are firm 1.

5 See Drazen and Sakellaris (1999).

6 Parameter $u$ and $v$ have a restricted set of values so that firms are more likely to receive positive signals if demand is high than if it is low. In the absence of these restrictions, signals would give the opposite information and investors would consider $u$ as $v$ and vice-versa.

7 In contrast, the closer probabilities $u$ and $v$ are to 0.5, the less information signals provide to investors. In the extreme, where both $u$ and $v$ are 0.5, signals are useless in that signals arrive with same probability regardless of the true demand. Consequently, $\pi_i = \pi_{i+1}$ for any $s$.

8 Other examples include Angeletos and Calvet (2006) and Meh and Quadrini (2004).

9 Given the large number of replications, the average reserve utility approaches its mean, which is what firms assume ex-ante about the other firm’s $r_0$.

10 Scharfstein and Stein (1990) find that under some conditions, firms may imitate the investment decisions made by others, “ignoring substantive private information.”

REFERENCES


